

Investigation of The Small Oscillations of Electrical Systems

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Abstract:

The article shows the efficiency of the joint application of the equations of nodal voltage and the Lyapunov's function in quadratic form for the analysis of small oscillations of electrical system. The jointly solution of the equations of nodal voltage and the Lyapunov's matrix equation makes it possible to determine the conditions for the stability of the electrical system and to reveal a generator that first approaches the stability limit. As will be shown below, studies of small oscillations of complex electrical systems can be carried out in full on the basis of matrix methods, successfully developed in recent decades. This is facilitated by the removal of memory limits and a sharp increase in the speed of computing modern computers.

Keywords:

Electric Power System, Nodal Voltage Equations, Lyapunov's Function, Small Oscillations, Sylvester's Criterion

1. Introduction

The growth of production in industry and services requires, as a rule, an increase in electricity consumption, which in turn depends on the commissioning of new generating capacities, power lines and substation equipment. New equipment installed in enterprises, and new household electric power plants presuppose higher quality electric power compared to what is being delivered to consumers in a number of districts at the present time.

The highest danger that disturbs normal power supply is the emergency mode in the electric power system (EPS), called a system accident. In such an accident, all equipment that uses electricity is stopped, which leads not only to underdevelopment, but also to huge losses from the marriage and damage to products, as well as to the accident of the manufacturing equipment itself. It is not difficult to understand that, given the substantial duration of such accidents, there may be human casualties.

It should be noted that system failures can occur unnoticed, for no apparent reason, during normal operation of the electrical system. Such accidents are associated with

the essence of energy systems, which are complex technical systems. A very dangerous and insidious property of electric energy is the absence of any visible signs of its appearance (due to an accident) on metal elements and details (or wet parts) with which a person normally touches. There is a need to ensure that in no case is such a confluence of circumstances permitted that would lead to a systemic accident.

The foundation of security is laid on every section of the power system and every electric consumer, including households. Here, the rules for the installation of electrical installations and the rules for their technical operation must be observed.

On the energy hierarchy, the correct electricity supply must be provided: generation, transportation and distribution of electricity in order to ensure its proper quality when consumed.

To solve these problems by the electric power system, it is necessary to constantly evaluate its static stability or resistance to "small" fluctuations, since it is the violations of such stability that lead to systemic accidents with their negative consequences. To assess the state, from the standpoint of static stability, there must be highly skilled specialists - engineers and masters who have received special training in the field of the theory of oscillations and who have the skills of performing complex calculations.

The problem of studying the stability of modern complex electrical systems is complicated in connection with the presence of weak bonds in their compounds, the presence of various regulating devices that impede the determination of the overall tuning, aggregates with different constants, etc. [1, 2]. This is also due to the inadequacy of studies that establish the degree of approximation to the truth not only of the results, but also of certain basic prerequisites of known calculation methods, since they all determine only necessary or sufficient conditions for the stability of electric power systems. At present, for solving practical problems, methods based on calculating the synchronizing power of one of the plant's power stations are used, i.e. which determines the aperiodic stability under the assumption of the absence of self-oscillation.

We will try to depart from the traditional method and apply the matrix method, based on the Lyapunov's functions in quadratic form, to solve the problem of static stability.

The wide inculcation of powerful and fast digital computers into the practice of dispatching and research calculations and, especially, the prospects for their development [1], remove the limitations on the use of more labor-intensive computationally effective but rigorous methods of stability analysis. These circumstances created good prerequisites for the application of the method of Lyapunov's functions in quadratic form for the analysis of small oscillations of complex EPS.

A rated analysis of small oscillations of EPS of varying complexity shows that the most rigorous theoretically, convenient for computations and effective by results is the use of two fundamental methods - the method of Lyapunov's functions in quadratic form and nodal equations [1, 4].

The methods for studying small oscillations with allowance for self-oscillation are complex; therefore sufficiently reliable results can be obtained with a rigorous mathematical description of the control system for controlled objects, using their reliable parameters and characteristics [7, 8].

2. Materials and Methods

The review of publications shows that the application of the matrix Lyapunov's and Riccati equations is expanding, as the role of matrix methods of investigation of linear dynamical systems increases, which is associated with the development of algorithms and programs of numerical and analytical methods for their solutions. Particular mention should be made of intensive studies leading to new and extremely effective methods of solving matrix equations (including analytical ones based on the technology of embedding systems - the canonization of matrices), and are widely used in practice.

It is known [2-4] that the Lyapunov's function in quadratic form for linear differential equations is the only one that provides necessary and sufficient conditions for the stability of the system under study when small perturbations arise in it. Therefore, the basis of the research in this paper is the Lyapunov's function in quadratic form and nodal equations, and the subject of research is the linearized differential equations of the elements of electroelectric systems. Matrix equations of the elements of electrical systems, which are the main part of electro-electric systems, are compiled on the basis of the equations of state variables that have obtained the greatest spread. They are small deviations of the mode parameters-the angles of the load of the rotors of synchronous generators, busbar voltages, power, and other regime parameters of electro-electric systems. The considered matrix equations are used for analysis of transient processes and static stability of electroelectric systems and synthesis of optimal parameters of regulators of synchronous machines operating in the electrical system.

The steady-state regime of the investigated EPS is determined on the basis of the equations of nodal voltages. Nodal equations, wearing a functional connection between the currents and voltages of nodes, most fully describe the electrical state of the network of any complexity [5].

In the general case, the nodal equations can be written in the form [5, 7]:

$$YU = I + Y_{i0}U_0 + J^*, \quad (1)$$

where

$$Y = \begin{bmatrix} y_{11} & -y_{12} & \dots & -y_{1n} \\ -y_{12} & y_{22} & \dots & -y_{2n} \\ \dots & \dots & \dots & \dots \\ -y_{n1} & -y_{n2} & \dots & y_{nn} \end{bmatrix}. \quad (2)$$

Conductivity matrix of the researching system; I , Y_{i0} , J^* - matrix-columns, or nodal currents, coupling conductors with a balancing node, current sources, which are transverse branches with given conductance.

To determine the stability, one can use the Lyapunov's function method in a quadratic form:

$$V(x) = x^T Q x. \quad (3)$$

The derivative of this function:

$$\frac{dV(x)}{dt} = \frac{d(x^T Q x)}{dt}. \quad (4)$$

Leads to the equation:

$$A^T Q + Q A = -C. \tag{5}$$

Equation (5), called the Lyapunov's matrix equation, ensures the condition that if the inequalities $V > 0$ and $\dot{V} < 0$ are satisfied simultaneously in some area of the space of variables (x_1, x_2, \dots, x_n) , including the origin, then the equilibrium position at the origin is asymptotically stable [3].

In (5) there appears an arbitrary positive definite matrix C , which is usually chosen as the unit matrix.

To solve the node equations, we choose the Newton's method in polar coordinates, to the advantages of which we can include the quadratic convergence of iterative processes, the possibility of further use for solving optimization problems and in calculating stability [5]. In addition, the voltages of the nodes U_j and the load angles of the generators δ_j , which are used in the Lyapunov's equations, determined on the basis of the solution of the node equations, contain all information about the state of the system, no matter how complex it may be [4].

On the basis of the calculated values of the voltage of the generator nodes and the nodes containing rotating machines, the positivity of the main diagonal minors of the matrix of the quadratic form (3) that establish the necessary and sufficient conditions for the stability of the generators (stations) and the EPS are consistently verified [6]. Essentially, the problem of analyzing small oscillations of a complex EPS is reduced to a multiple study of the "generator-bus" scheme [2].

In the classical case, the equations describing small oscillations in EPS are homogeneous linear (linearized) differential equations and have the form [4, 10]:

$$\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n; \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n; \\ \frac{dx_n}{dt} &= a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n; \end{aligned} \tag{6}$$

or in matrix form

$$\dot{x} = Ax, \tag{7}$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \tag{8}$$

and $x^T = [x_1, x_2, \dots, x_n]^T$ - transposed vector of state variables.

To determine the stability, we use Lyapunov's method and define it in the form of a positive definite quadratic form

$$V(x) = x^T Q x, \tag{9}$$

or $V = \sum_{i,j=1}^n q_{ij} x_i x_j$.

The derivative of this function is:

$$\begin{aligned} \frac{dV}{dt} &= \frac{d(x^T Q x)}{dt} = \left(\frac{dx}{dt}\right)^T Q x + x^T Q \left(\frac{dx}{dt}\right) = \\ &= (Ax)^T Q x + x^T Q Ax = x^T A^T Q x + x^T Q Ax = \\ &= x^T (A^T Q + QA)x. \end{aligned} \tag{10}$$

We require that the Lyapunov function satisfy the requirement

$$\frac{dV}{dt} = -W, \tag{11}$$

where $W = x^T C x$ is an arbitrary positive definite symmetric matrix.

Equating expressions (10) and (11), we obtain the equation:

$$A^T Q + QA = -C \tag{12}$$

Equation (12), called the Lyapunov's matrix equation, ensures the condition that if the inequalities $V > 0$ and $\dot{V} < 0$ are satisfied simultaneously in some region of the space of variables (x_1, x_2, \dots, x_n) including the origin, then the equilibrium position at the origin is asymptotically stable [4].

Note that both matrices Q and C are symmetric. Indeed, if the matrix Q is symmetric, that is, $Q^T = Q$, then

$$C^T = -(A^T Q + QA)^T = -Q^T A - A^T Q = -(A^T Q + QA) = -C \tag{13}$$

and, consequently, the matrix C is symmetric.

The functions (13), where Q is a positive definite symmetric matrix satisfying the conditions of the Lyapunov's theorem, are called quadratic Lyapunov's functions.

Lyapunov's theorem reduces the verification of the stability of the system under investigation to the solution of a linear matrix equation. Since the matrix Q is symmetric, the Lyapunov's equation is equivalent to the system of $n(n+1)/2$ linear algebraic equations. For a large dimension of the matrix A , the solution of such a system is less time-consuming than the calculation of the characteristic polynomial of the matrix A [4].

According to Sylvester's theorem [4, 9], the positivity of the principal diagonal minors of the matrix Q of the coefficients of the quadratic form (3) is a necessary and sufficient condition for the stability of the system under study for small perturbations:

$$Q = \begin{vmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{vmatrix} > 0,$$

i.e. $\Delta_{J11}=q_{11}>0, \Delta_{J12}=\begin{vmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{vmatrix} > 0, \dots \Delta_{J1n}=\begin{vmatrix} q_{11} & q_{12} & \dots & q_{1n} \\ q_{21} & q_{22} & \dots & q_{2n} \\ \dots & \dots & \dots & \dots \\ q_{n1} & q_{n2} & \dots & q_{nn} \end{vmatrix} > 0.$

Analysis of violation of the condition $\Delta_{J11}=q_{11}>0$ shows all kinds of violation of the stability of the electrical system (aperiodic violation, self-excitation, self-oscillation) [4, 12]. It is established that a violation of the stability of the electrical system is also detected for other principal diagonal minors, if $\Delta_{Ji} < 0$, where $i=1 \div n$.

We apply the Lyapunov function in quadratic form to study the static stability of a regulated electrical system. The main attention will be paid to the solution of the Lyapunov's matrix equation (12).

Let us first consider the compilation of equations in the deviations of the state variables of a regulated system of EPS. Suppose that the matrix A is stable, and the matrix C is positive definite.

The linearized equations of the simplest EPS with the presence of automatic regulators of excitation of proportional or strong action on a synchronous generator have the form [2, 4]:

- Equation of relative motion of the rotor of the synchronous machine:

$$T_j(d^2\Delta\delta/dt) = -P_d(d\Delta\delta/dt) - \Delta P.$$

- Equation of transients in the excitation winding:

$$T_{d0}(\Delta E'_q/dt) = \Delta E_{qe} - \Delta E_q.$$

- Equation in excitation winding:

$$T_e(\Delta E_{qe}/dt) = k_e \Delta e - \Delta E_{qe}.$$

- The equation of the amplifying element:

$$T_a(\Delta e/dt) = k_a \Delta u - \Delta e.$$

- Equation of the measuring element:

$$T_m(d\Delta u/dt) = k_m \Delta u_g - \Delta u.$$

- An equation that reflects the effect of an automatic excitation controller:

$$\Delta e = \sum_j (k_{0Pj} \Delta P_j + k_{1Pj} (d\Delta P_j / dt) + k_{2Pj} (d^2 \Delta P_j / dt^2)).$$

Here, T_j , T_{d0} , T_e , T_a , T_m are the aggregate's permanent inertia, the time constants, respectively, of the excitation windings with the stator winding open, exciter, amplifying element, transforming and measuring elements ($T_m = T_i$); $\Delta\delta$, $\Delta E'_q$, ΔE_q , ΔE_{qe} , Δe , Δu , Δu_g - deviations of the load angle, transient emf, emf. idling, emf. on the rotor rings, voltage on the exciter plates and voltage on the generator busbars; ΔP_j - parameters of the mode for which the excitation of the generator is controlled; P_d is the damper coefficient; k_{0Pj} , k_{1Pj} , k_{2Pj} - amplification factors on the channels of regulation of the automatic excitation controller, respectively - according to the deviation, according to the first and second derivatives of the regime parameters.

The deviations of the regulated parameter of the generator or system mode are determined by the relationship:

$$\Delta P_j = (dP_j/d\delta)\Delta\delta + (\Delta P_j/dE_q)\Delta E_q.$$

Two important results were obtained on the basis of the studies.

1. The stability condition of an electrical system for small deviations, which characterizes the positivity of the first minor of the matrix of the quadratic form $q_{11} > 0$ of the Lyapunov's function in quadratic form. This criterion is called simplified [11], since its positivity determines the positivity of the higher minors of the matrix. It is important that the condition $q_{11} > 0$ contains theoretically known types of violation of

the stability of the electrical system, and therefore contains both necessary and sufficient performance.

2. The condition for the approximation of the i -th generator to the limit is analytically determined under the conditions of a complex system containing n generators:

$$\frac{dq_{11j}}{dP} \rightarrow \max,$$

where P is any parameter.

3. Results and Discussion

Consider a simple power transmission system consisting of a generator, a transformer, a transmission line for the case of the presence of a strong action on the synchronous generator of the automatic excitation controller that reacts to the deviations and the first derivative of the angle ($\Delta\delta$) and the generator voltage (Δu_g). The time constants of the measuring and amplifying elements are taken into account under the condition $T_m = T_a$. Then the matrix of coefficients (8) has the form:

$$A = \begin{pmatrix} 0 & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 \\ a_{61} & 0 & a_{63} & 0 & 0 & a_{66} \end{pmatrix},$$

where $a_{12} = 1$, $a_{21} = -c_1/T_j$, $a_{22} = -P_d/T_j$, $a_{23} = -b_1/T_j$, $a_{32} = -(U(x_d - x'_d)/x'_{d\Sigma})\sin(\delta)$,

$a_{33} = -1/T_{d0}(dE'_q/dE_q)$, $a_{34} = 1/T_{d0}(dE'_q/dE_q)$, $a_{44} = -1/T_e$, $a_{51} = (k_{0\delta} + k_{0U}(du_g/d\delta))/T_a$,

$a_{52} = (k_{1\delta} + k_{1U}(du_g/d\delta))/T_a$, $a_{53} = k_{0U}(du_g/dE_q)$, $a_{55} = -1/T_a$, $a_{61} = -(du_g/d\delta)/T_m$,

$a_{63} = -(du_g/dE_q)/T_m$, $a_{66} = -1/T_m$, $c_1 = (E_q U/x_{d\Sigma})\cos(\delta)$, $b_1 = (U/x_{d\Sigma})\sin(\delta)$,

$dE'_q/dE_q = x'_{d\Sigma}/x_{d\Sigma}$, $\sin \delta_g = \sqrt{1 - (\cos \delta)^2}$, $du_g/d\delta = u_g(\sin(\delta_g)\cos(\delta) - \cos(\delta_g)\sin(\delta))$,

$du_g/dE_q = (x_s/x_{d\Sigma})\cos(\delta_g)$.

For $U = 1$, $P_d = 3$, $x_s = 0,3$, $x_{d\Sigma} = 2,3$, $T_j = 7\text{sec.}$, $T_{d0} = 2\text{sec.}$, $T_e = 1\text{sec.}$, $T_m = T_a = 0,1\text{sec.}$, $k_{0U} = 10$, $k_{1U} = 30$, $k_{0\delta} = 10$, $k_{1\delta} = 10$ the theoretical limit of the system under consideration with respect to the angle was $\delta = 152^\circ$.

Let us consider the joint application of the nodal voltage equations and the Lyapunov's function in quadratic form using the example of the 3-node scheme (Fig. 1).

The null node is taken as the basis, the first and third nodes are generating, and the second node is load.

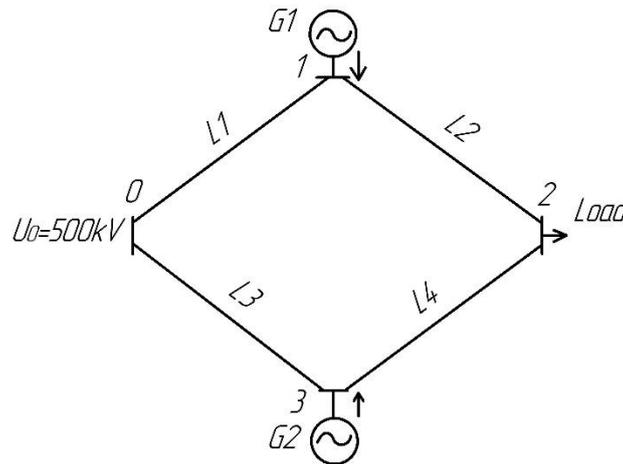


Figure 1. Schematic diagram of a three-site electrical system.

The analysis of the static stability of a complex EPS will be performed on the basis of known assumptions [1]:

- When calculating the synchronizing power of any of the generators, the angles of the rotors of all other generators remain unchanged. In this case, the capacities of all the system generators change;
- We will offer emf. generators constant for a given mode, the parameters of the circuit for replacing the electrical system and the loads are constant, while the active components of the complex resistances are not taken into account ($r = 0$);
- In the steady mode of operation of a complex power system, the machines can be expressed through the intrinsic and mutual conductances of the branches of the electrical system replacement circuit, which are also considered constant.

As is known, to study the static stability of complex systems use the positional mathematical model of EPS, which has the form [1, 6]:

- System of differential equations of relative motion of rotors of synchronous generators:

$$T_{j1} \frac{d^2 \delta_1}{dt^2} + P_{d1} \frac{d\delta_1}{dt} = P_{10} - P_1(\delta_{12}, \delta_{13}, \dots, \delta_{1n});$$

$$T_{j2} \frac{d^2 \delta_2}{dt^2} + P_{d2} \frac{d\delta_2}{dt} = P_{20} - P_2(\delta_{12}, \delta_{13}, \dots, \delta_{1n});$$

.....

$$T_{jn} \frac{d^2 \delta_n}{dt^2} + P_{dn} \frac{d\delta_n}{dt} = P_{n0} - P_n(\delta_{12}, \delta_{13}, \dots, \delta_{1n}).$$

- equations of powers of synchronous generators, expressed through the intrinsic and mutual conductivities of the branches of the substitution circuit:

$$P_1 = E_1^2 y_{11} \sin \alpha_{11} + E_1 E_2 y_{12} \sin(\delta_{12} - \alpha_{12}) + \dots + E_1 E_n y_{1n} \sin(\delta_{1n} - \alpha_{1n}),$$

$$P_2 = E_2^2 y_{22} \sin \alpha_{22} + E_1 E_2 y_{12} \sin(\delta_{12} - \alpha_{12}) + \dots + E_2 E_n y_{2n} \sin(\delta_{2n} - \alpha_{2n}),$$

.....

$$P_n = E_n^2 y_{nn} \sin \alpha_{nn} + \sum_{i \neq j}^n E_i E_j y_{ij} \sin(\delta_{ij} - \alpha_{ij}),$$

where δ_i and δ_{ij} are the absolute and relative load angles of the generators; E_i - electromotive forces of generators; T_j - constants of inertia of aggregates; P_{di} - equivalent damper coefficients of generators; P_i - electromagnetic power of synchronous generators; y_{ii} , y_{ij} - proper and mutual conductivity of the system; α_{ii} and α_{ij} are the corresponding complementary angles.

With respect to the above systems of equations, we solve the equation of the Lyapunov function in the quadratic form (5).

Below are the initial data and parameters of a complex electrical system (Fig. 1).

Generating node parameters:

G1: $P_1=100$ MVt; $\cos\varphi_1=0.8$; $U_{G1}=500$ kV; $T_{j1}=6$ sec.; $x_{d1}=1.907$; $x'_{d1}=0.278$.

G2: $P_2=60$ MVt; $\cos\varphi_2=0.8$; $U_{G2}=500$ kV; $T_{j2}=5.4$ sec.; $x_{d2}=1.915$; $x'_{d2}=0.275$.

The nodes are connected among themselves by the respective overhead power lines L1-L4.

Parameters of power lines:

L1: $U_{L1}=500$ kV; $\ell_{L1}=95$ km; $r_0=0.0397$ Ohm/km; $x_0=0.31$ Ohm/km.

L2: $U_{L2}=500$ kV; $\ell_{L2}=115$ km; $r_0=0.0362$ Ohm/km; $x_0=0.306$ Ohm/km.

L3: $U_{L3}=500$ kV; $\ell_{L3}=80$ km; $r_0=0.0397$ Ohm/km; $x_0=0.31$ Ohm/km.

L4: $U_{L4}=500$ kV; $\ell_{L4}=75$ km; $r_0=0.0397$ Ohm/km; $x_0=0.31$ Ohm/km.

Load node parameters:

$P_{Load}=150$ MVt; $\cos\varphi_{Load}=0.88$; $U_{Load}=500$ kV.

The weighting of the regime is carried out gradually by increasing the active and reactive load of the second node. The generators are equipped with automatic excitation controllers of strong action, responsive to the deviations and acceleration of the angle change, as well as to voltage deviations.

The calculation of the steady-state regime is carried out and the positivity of the first minor q_{11} of the matrix of the quadratic form Q is tested for generating nodes.

Figure 2 shows the change in the first diagonal minor q_{11} of the matrix Q of the Lyapunov's function in the quadratic form.

The calculation is made with weighting of the regime - a gradual increase in the load $P_{Load} = 150$ MVt to $P_{Load\ max} = 200$ MVt, which leads to an increase in the angle to $\delta_{cr} = 138^\circ$.

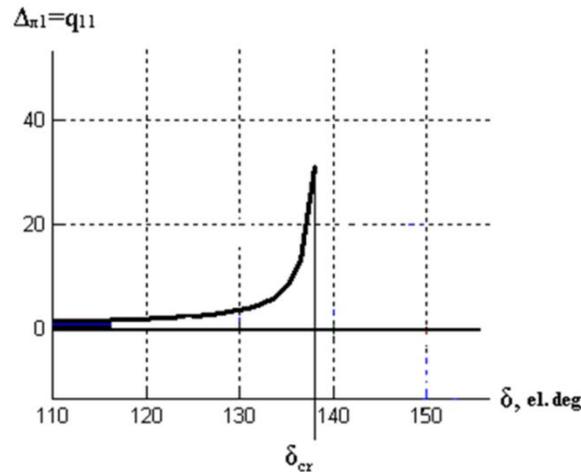


Figure 2. The character of the change of the first diagonal minor q_{11} of the matrix Q of the Lyapunov's function in the quadratic form.

4. Conclusions

The significance of this result for the theory of small oscillations in the electrical system and the practice of operating complex electrical systems is obvious, because as a whole, the stability of the electrical system is determined for $q_{11} > 0$ and a generator approaching the stability limit at $\frac{dq_{11j}}{dP} \rightarrow \max$.

A distinctive feature of this approach was the resolution of the equations of state of the system relative to absolute angles, in contrast to the traditional equations of the system compiled with respect to the mutual angles [1, 6]. This approach simplifies the study of transient modes of electrical systems, including in the analysis of their static stability.

It should be noted that when perturbations occur in the electrical system, the loss of stability occurs as a result of the synchronous generator output from synchronism or rotating machines in the general case. Static elements also affect the stability of EPS by its parameters, which are usually taken as constant or slowly changing. Therefore, the main task is to determine the conditions for the output from synchronism of a particular synchronous generator or their groups (stations) in a complex EPS. The method of Lyapunov's functions in quadratic form allows us to solve such a problem.

In the opinion of the authors, studies of small oscillations of the electrical system based on Lyapunov's functions in quadratic form should be developed and conducted in the directions:

- Improvements and selection of the model of nodal equations for joint application with the Lyapunov's function in quadratic form;
- Development of a more accurate model of the electrical system;
- Development of matrix methods for the aggregate and interconnected optimal control between units and stations of the electrical system;
- Development of an algorithm and model for optimal control, evaluation and synthesis of the relevant EES control laws for the probabilistic nature of the initial information;

- Analysis of control action models for the introduction of post-emergency regimes into the stability region.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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