Mathematical Model of the Korotkoff Sounds

Yuriy N. Zayko[1*]
[1] Department of Applied Informatics, Faculty of Public Administration, Russian Presidential Academy of National Economy and Public Administration, Stolypin Volga Region Management Institute, Saratov, Russia

Email Address

zyrnickl@rambler.ru (Yuriy N. Zayko)
*Correspondence: zyrnick@rambler.ru

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Abstract:

In this article briefly describes the main areas of research of the Korotkoff sounds accompanying the blood pressure measurement process. It is noted that the existing approaches, unlike the approach of the author do not sufficiently involve the achievements of modern hydrodynamics. This applies in particularly to the study of flows in tubes with elastic walls. A brief overview of the author's works in this area is presented. Based on these results, a model describing the Korotkoff sounds is presented. This model is based on the equation derived from the Navier-Stokes equations by the method of multiscale decompositions. Its solution in the form of a shock wave with a perturbed front is explored. We find the equation for the frequency of the shock wave front vibrations. The stability of a one-dimensional shock wave with respect to small perturbations is also investigated. It is concluded that the blood flow acquires a more complex transverse structure with an increase in the heart rate. This phenomenon will help to understand the turbulence development in the blood flow, for example, with atrial flutter.

Keywords:

Blood Flow, Blood Pressure, Navier-Stokes Equations, Tubes with Elastic Walls, Korotkoff Sounds, Shock Wave

1. Introduction

Korotkoff sounds (KS) are accompanying the blood pressure measurement process by a tonometer. It is believed that a complete physical description of their mechanism is absent, although, in principle, this mechanism seems quite clear, and is caused by pushes of flow according to change of effort which is squeezing the air from the tonometer cuff [1]. For the construction of a mathematical model of the phenomenon one must take into account, at least three factors - flow nonlinearity, diffraction phenomena associated with the presence of the vessel walls and the flow viscosity. Non-Newtonian nature of the blood is usually is not taken into account [2].

There are many theoretical models of KS. As mentioned in the lecture of A. Tsaturyan [3] "It is almost as popular to invent the theories of Korotkoff sounds as proving Fermat's theorem." We do not set a goal to provide a comparative overview of all areas of KS research. Instead, we consider only two of the most famous approaches:

A. The approach, which links the occurrence of KS with vibrations of a clamped artery wall.

B. The approach, which explains the occurrence of KS due to shock waves in the bloodstream.

Biophysical approach within both of these directions is presented in Refs [1 - 9]. Noteworthy is the fact that the achievements of hydrodynamics in this area are not used sufficiently by the authors. In particular, is no discussion of analogy between Korotkoff sounds and the sequence of solitons in the channel which was blocked by partition after its sudden removal [10]. This leads to the fact that the results obtained in the framework of approach mentioned, have little to do with the basic provisions of hydrodynamics and seem ill-founded. In contrast to these works, the author in the articles [11 - 16] developed a physical approach to the issues.

In the article [11], based on the Navier-Stokes (NS) equations for viscid fluids, supplemented by state equation binding the pressure and density of the fluid including the nonlinear terms with the help of multiscale perturbation theory the multi-dimensional generalization of the Burgers equation [17] was received for the case of the predominance of nonlinearity in comparison with diffraction effects. This equation differs from the known equation of Zabolotskaya-Khokhlov-Kuznetsov (ZKK) [18] obtained for the opposite case. In the article [12] were investigated previously unknown solutions of these equations in the form of shock waves, on the front of which the secondary waves propagate.

In the articles [13-14] it was considered a flow of one-dimensional inviscid fluid in a tube with elastic walls. The study was based on the so-called model of local response [19], supplemented by the equation of the vibrations of wall considered as an elastic shell [20]. The flow stability was investigated and was found the boundaries of the convective and absolute instabilities. To describe the flow of a fluid was used the two-fluid model which was used earlier to describe the super fluidity in liquid helium [21]. Abnormally high (in two orders of magnitude) value of the critical Reynolds number for flows in tubes of round profile was explained [21]. The papers [15, 16] are devoted to the description of vortex formation in that flow.

The results of these articles are directly related to the issues of this article.

Considering the above, it is premature to make a final decision in favor of one or another model of KS description. Most likely, as is often the case when describing the complex phenomena occurring in living organisms, none of the proposed models can qualify for a full description of the phenomenon.

Next, we follow the work [11]. Before, let’s say a few words about the different approaches in the framework of the chosen direction. In the literature (see [18] and references therein) there is the misconception that the two different cases corresponding to the predominance of the effects of diffraction over nonlinear effects and the opposite case can be described by the one and the same model equation ZKK. In fact, each case is described by its own equations, which both are derived from the NS equations with the help of multiscale expansions but in different conditions. Accordingly, the equations for describing the transverse perturbations on the front of the shock wave one of which is used below are also different; though look similar in shape [12].

2. Basic Equations

The original equation obtained in the paper [11] looks as follows (below is presented the equation ZKK for comparison)

$$
\begin{aligned}
&(2p_\tau + \alpha p p_\xi - \sigma p_{\xi\xi})_\tau - c^2 \Delta_\perp p = 0 \\
&(p_\xi + \alpha p p_\tau - \sigma p_{\tau\tau})_\tau - c \;\; \Delta_\perp p = 0 \qquad ZKK
\end{aligned} \tag{1}
$$

Here p – is an excessive pressure above some average value p_0 for the flow, ξ – longitudinal (along the flow) stretched coordinate, τ – slow time, $\Delta_\perp$ - transverse Laplacian, c – speed of sound in a flow; the coefficients α and σ are uniquely determined by the method of multiscale expansions of the original Navier-Stokes equations: α depends on the nonlinearity and σ – on the viscosity of the flow. In the article [12] it is shown that equation (1) has an exact solution in the form of a shock wave with a perturbed front

$$
\begin{aligned}
&p = p_2 + a\left\{1 + \exp\left[\frac{\alpha a}{2\sigma}(\xi - \varphi(\tau,\eta,\zeta))\right]\right\} \\
&a = p_1 - p_2,\, p_1 = p(\xi = \infty),\, p_2 = p(\xi = -\infty)
\end{aligned} \tag{2}
$$

η, ζ – are the coordinates transverse to the flow direction. The function $\varphi\ (\tau,\ \eta,\ \zeta)$ is satisfied to the equation

$$
\varphi_{\tau\tau} - \frac{c^2}{2}\Delta_\perp \varphi = 0 \tag{3}
$$

Solutions of the equation (3) describe the perturbation of the shock wave front. Equation (3) is supplemented by the boundary condition $\nabla_\perp \varphi\,|_{\gamma=0} = 0$, where $\gamma(\eta,\zeta)=0$ – is an equation of border of the vessel wall, which is assumed to be absolutely rigid. If we put $\varphi \equiv \varphi(\tau) = V\tau, V = \frac{\alpha}{4}(p_1 + p_2)$, then the solution of the equation (2) describes the well-known one-dimensional shock waves with finite front width propagating along a stream at a rate V [17].

Equation (3) is a known equation of a vibrating membrane, which solutions are well known and look as follows [22]:

$$
\varphi(\tau, r, \theta) = J_m(kr)\left(Ae^{im\theta} + Be^{-im\theta}\right)\left(C\cos\omega\tau + D\sin\omega\tau\right) \tag{4}
$$

where J_m – is the Bessel function of order m, $k = \omega\sqrt{2}/c$, ω – vibration frequency, $r = \sqrt{\eta^2 + \zeta^2}, \theta$ -are the radial and angle coordinates in the membrane plane; A,B,C,D- are constants which are determined from initial conditions. The most interesting is the solution which is not depending on θ, corresponding to the value of m = 0. The boundary condition on the vessel wall of radius ρ leads to the equation which permits to find the frequency of vibration – $J_1\,(k\rho)$=0.

Let us make some estimation. If one takes for the average values of inner diameter of artery the value of 2ρ = 1 Sm [1], heart rate f = 70 Sec^{-1} and for the first three magnitudes of the roots J_1 [23]: R_1 = 3,831, R_2 = 7,015 and R_3 = 10,173 then he receives, correspondingly, three values of the speed of sound in artery c = 81,165; 44,324 and 30, 565 (Sm/Sec). These are little less in the order of value than the experimental data ($9 \div 1$ M/Sec) [1]. Figure 1 depicts the dependence of the speed of sound in an artery $c_n = 2\pi\sqrt{2} f\rho / R_n, n = 1,2,3$ on the magnitude of the heart rate f for the first three numbers of the roots J_1. It can be seen from this that with the increase in the heart rate, the calculated speed of sound falls within the experimental range. From Figure 1 it follows that in the blood flow in a blood vessel with an increase in the heart rate, areas with a more complex transverse structure may appear.

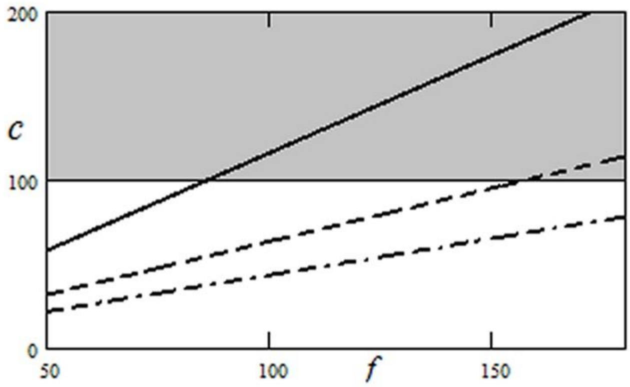

Figure 1. *The dependence of the speed of sound in artery c (sm/sec) on the magnitude of the heart rate f (1/sec) for the different number n of the roots J_1. The gray area shows the lower part of the range of experimental values of c.*

Solid line: n = 1, dashed: n = 2, dot-dashed: n = 3

3. Stability of the Shock Wave

The solution found is of interest only if the basic one-dimensional shock wave $P_{sh}(u)$ is stable. This section is devoted to the study of this. We represent the solution of equation (1) in the form [11]

$$p(\xi,\eta,\zeta,\tau) = P_{sh}(u) + \sum_j p_j(\xi,\eta,\zeta)e^{-\lambda_j \tau}, u = \xi - V\tau,$$
$$P_{sh}(u) = p_2 + \frac{a}{1 + \exp\left(\frac{\alpha a}{2\sigma}u\right)} \quad (5)$$

where the second term in (5) representing the perturbation is much smaller than the first. An important condition is its boundedness in ξ. Substituting (5) in the first equation in (1) and linearizing it we receive in the first order of smallness the equation for every values j

$$2\lambda_j^2 p_j + \alpha(P_{sh})_\tau (p_j)_\xi + \alpha P_{sh}(p_j)_{\xi\tau} - \sigma(p_j)_{\xi\xi\tau} - c^2 \nabla_\perp p_j = 0 \quad (6)$$

where sub-indices mean partial derivatives. We pass into the reference frame in which the shock wave is at rest, where $p_1 + p_2 = 0$, then the second term in the formula (6) vanishes. Representing the perturbation p_j in the form

$$p_j(\xi,\eta,\zeta) = q_j(\xi)\exp\left[i\left(k_\eta \eta + k_\zeta \zeta\right)\right] \quad (7)$$

and substituting this expression into the equation (6) we receive an equation for q_j

$$(q_j)_{\xi\xi} - \frac{\alpha}{\sigma}P_{sh}(\xi)(q_j)_\xi + E_j q_j = 0$$
$$E_j = \frac{2\lambda_j^2 + c^2 k_\perp^2}{\sigma\lambda_j}, k_\perp^2 = k_\eta^2 + k_\zeta^2 \quad (8)$$

Here $k_\perp(k_\eta, k_\zeta)$ is a transverse wave vector. Introducing new function $s_j(\xi)$

$$s_j(\xi) = q_j(\xi)\exp\left[-\frac{\alpha}{2\sigma}\int^{\xi} P_{sh}(\xi)d\xi\right] \quad (9)$$

we receive the equation for $s_j(\xi)$

$$(s_j)_{\xi\xi} + \left[E_j - U(\xi)\right]s_j = 0$$
$$U(\xi) = \left[\frac{\alpha}{2\sigma}P_{sh}(\xi)\right]^2 - \frac{\alpha}{2\sigma}P'_{sh}(\xi) \quad (10)$$

Performing calculations we get in the selected frame of reference $U(\xi) = \left(\frac{\alpha p_2}{2\sigma}\right)^2$,i.e. doesn't depend on ξ. The equation (10) looks like the well-known Schrödinger one. It is known that for $E_j > U$ its solutions do not grow with ξ . The condition of stability of a shock wave is $\lambda_j > 0$ that together with the condition of boundedness of the perturbation leads to the condition

$$c^2 k_\perp^2 > \frac{1}{128}\frac{(\alpha p_2)^4}{\sigma^2} \quad (11)$$

From the present analysis, it also follows that unstable solutions with $\lambda_j < 0$ are also unbounded in ξ and can't be investigated by this method.

It follows from the obtained condition that perturbations of the blood flow with large values $k_\perp$ and characterized by a more complex transverse structure have a greater degree of localization along the flow.

4. Discussion

Based on these results the Korotkoff sounds can be linked with vibrations of the shock wave front, discussed above. Explanation of the Korotkoff sounds using shock waves in the blood flow in the artery is also mentioned in Ref. [1]. That sounds normally alternate at equal intervals, thus indicating that one mode with a fixed frequency of vibration is realized. The emergence of additional frequency (arrhythmia) shows that the complex modes are arising.

Let us discuss another model of occurrence of the Korotkoff sounds. In Ref. [2] a model is constructed that takes into account the influence of the walls of the vessel which are presented as a passive elastic tube, on the character of the blood flow. An expression was received which binds the frequency of the Korotkoff sounds with local parameters of the vessel: an effective modulus of elasticity of the vessel wall, its thickness and an average diameter of the tube. These parameters are not constant along the vessel, so we can only speak about some accordance of the found frequencies and the frequency of the Korotkoff sounds averaged over the length of the vessel.

The model described in this paper relates the frequency of the Korotkoff sounds with phenomena which do not depend on the state of the vessel walls and are determined only by blood flow parameters: its speed, density, and viscosity. The presence of the vessel walls only affects the appearance of the original equation, which takes into account the diffraction in addition to the inertia and nonlinearity.

Despite the fact that the study of Korotkov's sounds lies in the mainstream of the general studies of blood flow in the blood vessels, in the last time interest in the origin of this phenomenon has somewhat weakened, although it is widely used in medical practice for the purposes of diagnosis [24]. In recent years, much attention has been paid to the numerical study of fluid flows in one-dimensional channels with elastic walls and their possible applications in medicine [25, 26]. A more realistic 3D case is also numerically investigated [27, 28]. The last papers focus on the study of non-Newtonian character of the blood flow. One can express the hope that the latest results of blood flow studies will provide a complete picture of the origin of Korotkoff sounds.

5. Conclusions

In this article, the results of the investigation of the so-called Korotkoff sounds are presented. The results are based on the fundamental approaches and models used in the hydrodynamics and nonlinear physics, such as the Navier-Stokes equations, multiscale decompositions, etc. This differs the present results from the results of other works which have special modeling sense. Some comparison of the theoretical calculations with experimental data is presented. It can be noted a rather good agreement (in the order of value) of this comparison.

The stability of a one-dimensional shock wave with respect to small perturbations is also investigated. It was shown that with an increase in the heart rate in the bloodstream, sections with a more complex transverse structure appear. This phenomenon will help to understand the turbulence development in the blood flow, for example, with atrial flutter.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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