

The Hyperbolic Universe Does Not Need Dark Energy

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Abstract:

Although the perspective for nearby objects in the hyperbolic space is very nearly identical to Euclidean space (i.e. the Universe locally is approximately flat consistent with local observations), the apparent angular size of distant objects falls off much more rapidly, in fact exponentially. The Universe globally is hyperbolic as we did prove mathematically. The current observed density of the Universe $\rho = 10^{-31} \text{g/cm}^3$ is consistent with a hyperbolic open universe. The hyperbolic universe consists of zero cosmological constant. The hyperbolic universe doesn't need Dark Energy to account for the accelerated expansion. The Hyperbolic Universe grows exponentially preserves a legitimate inflation covers the current observed large structure $1.3 \times 10^{28} \text{cm}$. The equation of state of cosmology $p = -\rho$ is a property of the hyperbolic structure of the hyperbolic universe. We calculated the Hubble constant due to the Hyperbolic Universe, $H = 72.34 \text{ [(Km/sec)/Mpc]}$.

Keywords:

Hyperbolic Universe, Modified Gravity, Dark Energy, Cosmological Constant, Quintessence

1. Introduction

The 1990's two teams of astronomers (Riess et al. 1998, Perlmutter et al. 1999), the Supernova Cosmology Project (Lawrence Berkeley National Laboratory) and the High-Z Supernova Search (international) were looking for distant Type Ia supernovae in order to measure the expansion rate of the Universe with time. They expected that the expansion would be slowing, which would be indicated by the supernovae being brighter than their redshifts would indicate. Instead, they found the supernovae to be fainter than expected from a uniformly expanding universe. Hence, the expansion of the Universe was accelerating! The idea of dark energy is as old as general relativity itself. Albert Einstein included it when he first applied relativity to cosmology exactly 100 years ago. Einstein mistakenly wanted to exactly balance the self-attraction of matter by anti-gravity on the largest scales. He could not imagine that the Universe had a beginning and did not want it to change in time. Almost nothing was known about the Universe in 1917. The very idea that galaxies were objects at vast distances was debated. Einstein faced a dilemma. The physical essence of his theory, as

summarized decades later: Matter tells space how to curve, and space tells matter how to move. That means space naturally wants to expand or contract, bending together with the matter. It never stands still. This was realized by Alexander Friedmann who in 1922 kept the same ingredients as Einstein. But he did not try to balance the amount of matter and dark energy. That suggested a model in which universes that could expand or contract. Further, the expansion would always slow down if only matter was present. But it could speed up if anti-gravitating dark energy was included. Since the late 1990s many independent observations have seemed to demand such accelerating expansion, in a Universe with 70% dark energy. But this conclusion is based on the old model of expansion that has not changed since the 1920s.

Cosmology, the study of the universe at its largest scales, is undergoing a period of intense development based on both rigorous theoretical and experimental input. The dominant force in cosmology is gravity. Einstein's theory of General Relativity, our modern theory of gravity, forms a solid theoretical underpinning of the recent progress. On the experimental side the discovery by Hubble of the expanding universe is the basis of the Big Bang model of a dynamic universe that allows a meaningful discussion of a beginning, and thus an age of the universe. The discovery of the Cosmic Microwave Back ground (CMB), the relic sea of photons left over from the early universe, further supports the Big Bang model. The realization from the rotation curves of galaxies that there is considerably more matter with attractive gravity, called dark matter, than we can see via electromagnetic radiation (the visible matter) complicates things. With attractive gravity between the components of the universe, the expectation was that the expansion velocity of the universe, the so called Hubble velocity, would decrease with time, i.e. a decelerating universe. *Note that:*

The critical density of the flat universe $\rho_c = 3H^2/8\pi G = 10^{-29} \text{g.cm}^{-3}$ corresponding to

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2} = 1.$$

The current observed density of the Universe $\rho = 10^{31} \text{g/cm}^3$, corresponding to $\Omega_{\text{baryonic}} = .04$, consistent with hyperbolic open universe. Dark Matter density corresponding to $\Omega_{\text{dark matter}} = 0.22$. Dark Energy density corresponding to $\Omega_{\text{dark energy}} = 0.74$

The observational problem is to discover objects that can be seen at large redshifts, so the cosmological effects are large enough to measure, and that are well enough understood so that their apparent brightness can be trusted to give a reliable measure of their distance. The long, winding path of observational cosmology is littered with the wreckage of past attempts to do this with galaxies, whose properties evolve over time much too rapidly to serve as "standard candles" for this work. But type Ia supernovae (SN Ia) can be seen to redshift 1, and their intrinsic scatter in brightness is small enough so that the cosmological effects on the observed brightness as a function of redshift can be measured. At a redshift of 0.5, the difference in apparent magnitude between a universe that is flat, decelerating, and just barely closed by matter, $\Omega_m = 1$, and a universe that is hyperbolic and empty, $\Omega_m = 0$, is $\approx 25\%$ in the flux of a supernova. The scatter in SN Ia brightness for a single object, after correcting for the light curve shape (as described below), is only $\approx 15\%$, so a relatively small number of supernovae can produce a significant measurement of the cosmology [1]. The result is surprising evidence for an accelerating, but *geometrically flat*, universe. The recent discovery that the Hubble velocity is increasing with time, an accelerating universe, was an immense surprise. Assuming the existence of dark matter and that the law of gravitation is universal, two teams of astrophysicists at the Lawrence Berkeley

National Laboratory found that the supernovas weren't brighter—and therefore nearer—than expected. They were dimmer— that is, more distant. The two teams both concluded that the expansion of the universe isn't slowing down. It's speeding up. This led to the postulation of the existence of a new component of the universe, dark energy, with some very unusual properties, not the least among which is that it has, in some sense of the word, repulsive gravity that drives the acceleration. The best estimates that fit all of the presently existing data is that the energy/matter density of the universe is made of, 4% of visible matter, dark matter making up 24% and dark energy making up the majority 72%. "Supernovae (SN) are extremely luminous explosions of dying stars. This makes them directly observable even at very far distances. Classification of SN was originally done spectroscopically, but even this simple identification tells a lot of information about the star's evolution and final explosion. The most important type and the one relevant for cosmological measurements are supernovae Ia. These explosions happen from collapsing white dwarfs in close binary star systems. The explosion is triggered when white dwarf reaches the Chandrasekhar mass limit in process of accretion from the binary companion. Since the limit has little varying value, all SN Ia are considered to have quasi-equal peak absolute magnitude. They are successfully identified both by spectrum and by the light curve. Luckily SN Ia is the most luminous and the most frequent type of supernovae explosions in the universe. By measuring their apparent magnitude, and joint with the redshift, this becomes a powerful method for sampling luminosity distance versus redshift relation. That function depends on cosmological model and can be used to constrain cosmological density parameters. Under standard model, the measurements reveal accelerated expansion of the universe that is explained by contribution of dark energy[2]. The Type Ia supernova as standard candles can be used to measure the expansion history of the universe i.e. the plot of the scale factor of the universe when the supernova light was emitted versus the time in the past when the supernova explosion occurred. This is done by measuring the apparent brightness L of the supernova and its redshift z . Comparing the apparent brightness to the presumably known intrinsic brightness of the supernova determines its distance and from the distance the time in the past when the supernova exploded can be inferred knowing the velocity of light. The redshift gives the scale factor of the universe at the time of the supernova explosion via $a(t)=1/(1+z)$. Each supernova then yields a point in the $a(t)$ versus t plot, and a large sample of supernovae thus measure the expansion history of the universe. Given the high level of interest in understanding the underlying reasons for the acceleration of the universe and the nature of dark energy, a vigorous program is planned worldwide to measure the cosmological parameters to a higher level of precision. In particular there is a focus on the measurement of the equation of state parameter w , which can distinguish between a cosmological constant ($w=-1$) or some other form of dark energy with $w<-1/3$ but not exactly -1 . The present dependence of w , which would be inconsistent with a cosmological constant, is very poorly constrained by the present data. Its results, $w=-0.93\pm0.16$ is consistent with the cosmological constant but higher precision may yet show a deviation from -1 . The parameter w_a , indicating a time interesting to point out that if it turns out that dark energy is due to a nonzero cosmological constant, a value of Ω_Λ in the vicinity of 0.7 would be implied. Such a small value of Ω_Λ would lead to one of the most dramatic inconsistencies in modern physics called the cosmological constant problem [3]. Modern field theory of particle physics predicts that the energy density of the vacuum contributes to the cosmological constant with a value 120 orders of magnitude larger than the experimentally observed value.

Actually this problem would persist even if the dark energy turned out to be something else than the cosmological constant, with $\Lambda=0$. Even though many theorists have tried, there is no explanation insight to this fundamental disagreement. Accelerating, is ruling out a nonaccelerating universe and a zero cosmological constant with a very high level of confident. All of the surveys are consistent with $w=-1$, assuming a constant w and a flat universe. "The accelerating expansion of the universe was discovered when astronomers were doing research on type 1a supernova events. Because all type 1a supernova explosions are remarkably similar in brightness, if we know how bright a star should be, we can compare the *apparent* luminosity with the *intrinsic* luminosity, and we get a reliable figure for how far any given object is from us. Incidentally, along with helping us make these key determinations about the locations of objects in the universe, these supernova explosions also gave us a sneak preview of one of the strangest observations ever made about the universe. To measure the approximate distance of an object, like a star, and how that distance has changed, astronomers analyze the spectrum of light emitted. Scientists were able to tell that the universe is increasing in expansion because, as the light waves make the incredibly long journey to Earth —billions of light-years away— the universe continues to expand. And as it expands, it stretches the light waves through a process called "redshifting" (the "red" is because the longest wavelength for light is in the red portion of the electromagnetic spectrum). The more redshifted this light is, the faster the expansion is going. Many years of painstaking observations (made by many different astronomers) have confirmed that this expansion is still ongoing and increasing because (as previously mentioned) the farther away an object is, the more redshifted it is, and (thus) the faster it is moving away from us [4]. Anyone with even a passing interest in space science is familiar with this unforeseen development that occurred in 1998. In that year the research into the decay of distant supernovae events revealed that these objects are actually considerably farther from us than had been expected. The findings were so contrary to theory that at first there was considerable doubt. It was only after another study, by an independent team, came to the same conclusion (that these "standard candle" exploding stars were 20% to 25% farther than expected) that the crisis hit home. The standard theories of the universe, the *hot big bang* model and *inflationary big bang* model, had predicted that the matter in the universe thrown out by the "big bang" should decelerate as gravity acts to slow down this matter and eventually pull it all inward in a "big crunch." But now, it seems, the opposite is true: the speed of expansion of the universe is increasing! Mysteriously, the universe is now accelerating— or so it is believed. And so, in 1998 astronomers and physicists convinced themselves that the universe is accelerating—getting bigger and bigger, faster and faster. Now there is a new mystery! What's driving this thing? Why is the universe accelerating? You can coast along on "expansion" but you need a force when you bring in "acceleration." No exception. Forget the minimal mystery of why the universe is expanding in the first place; now there is the utterly baffling mystery of why it is expanding with a vengeance! *In other words this new dark [acceleration leads to dilution which leads to darkness] universe is utterly baffling, a road map to new mysteries. Dr. Marc Davis, a cosmologist at the University of California at Berkeley, called it 'a universe chockfull of exotics that don't make sense to anybody.* [5].



Figure 1. Type 1a supernova depicted here at the lower left hand corner. Image credit: High-Z Supernova Search Team, HST, NASA [6].

Current models of the universe require dark energy to explain the observed acceleration in the rate at which the universe is expanding. Scientists base this conclusion on measurements of the distances to Type 1a supernovae in distant galaxies, which appear to be farther away than they would be if the universe's expansion was not accelerating. On the even larger cosmic scales of an expanding universe, gravity appears to be weaker than expected in a universe containing only normal matter and dark matter. Scientists invented a new force, called "dark energy", a sort of anti-gravitational force causing acceleration in the expansion of the universe out from the big bang 13.8 billion years ago.

Dark energy isn't noticeable on small scales, but becomes the dominating force of the universe on the largest cosmic scales: almost four times greater than the gravity of normal and dark matter combined. Locally the spacetime is flat, there is no cosmological redshift. For example, observations tell us that space within galaxies, which are rather diffuse objects, do not expand. Thus, where is the "borderline" in space which divides expanding space from non-expanding space? Two galaxies within our Local Group, including Andromeda, and a few galaxies in the Virgo Cluster display blue shifts and so are moving toward us, but these results from their local motion (peculiar velocity). Why nearby galaxies exhibit blue-shift? Because its peculiar velocity is greater than its recession velocity! The answer is more convenient if we say: *Locally the spacetime is flat through which the curvature is negligible* (no cosmological redshift) where the random peculiar velocity dominates. If cosmological redshift has nothing to do with the Doppler Effect, how do we know that galaxies that are very far away are also receding from us? How to compare between two unrelated concepts, the Doppler redshift and the cosmological redshift? Andromeda galaxy is blueshifted because it's sufficiently nearby where the spacetime is approximately flat and special relativity works. Its blue shifted according to the Doppler Effect in flat spacetime.

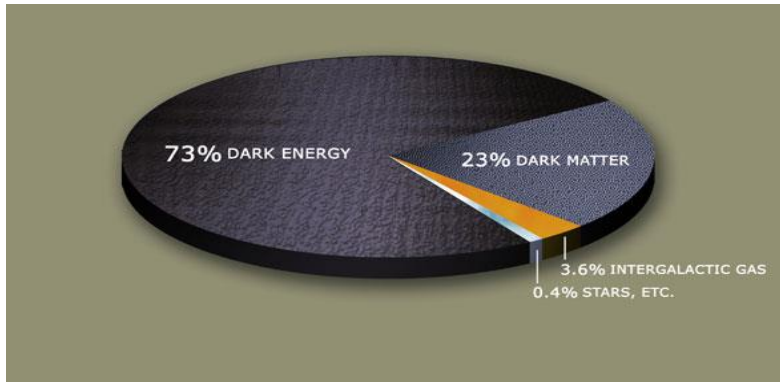


Figure 2. The best estimates that fit all of the presently existing data is that the energy/matter density of the universe is made of, 4% of visible matter, dark matter making up 23% and dark energy making up the majority 73%. [7].

The luminosity L of a nearby star can be determined from its apparent brightness f and distance determined by triangulation using the flat universe inverse square law, gives a predicted connection between the flux f and the redshift z

$$f = \frac{L}{4\pi d^2}$$

$$\therefore z = H_0 d$$

$$\frac{f}{L} = \frac{H_0^2}{4\pi z^2}$$

Those are appropriate approximations for nearby sources from which light takes only a short time to reach us, traveling a distance that is small compared to that over which space might be curved and small enough that Hubble's law holds. However, for standard candles that are further away, deviation from the previous equation can be expected, arising from *the spatial curvature* of the universe. Deviations can also be expected if the light from a standard candle travels to us over a time during which *the expansion of the universe* is significant.

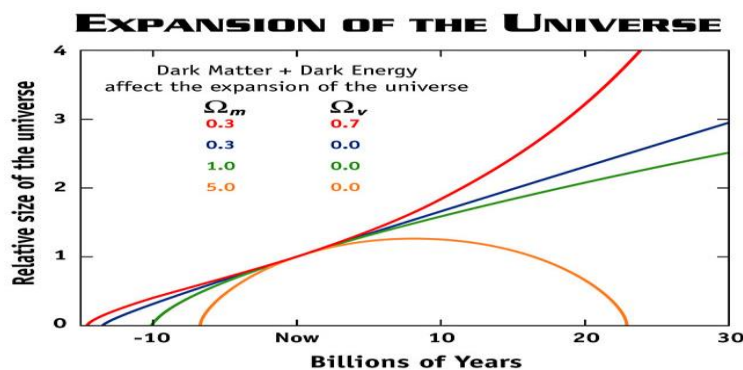


Figure 3. Scientists used to think that the universe was described by the yellow, green, or blue curves. But surprise, it's actually the red curve instead [8].

In addition, measurements of the cosmic microwave background indicate that the Universe has a flat geometry on large scales. Because there is not enough matter in the Universe either ordinary or dark matter to produce this flatness, the difference must be attributed to a "dark energy".

2. Searching Dark Energy

2.1. Equation of State

The consequences of dark energy for fundamental physics will not be clear until its origin is discovered, but the effects on the universe are dramatic. Dark energy effectively contributes 70-75% of the current energy density of the universe, governing the expansion of space, causing it to accelerate over the last ~5 billion years, and will determine the fate of the universe. Such a phenomenon is not predicted within the standard model of particle physics nor within experimental experience of gravity as an attractive force. Gravitation as an attractive force acts to slow down the cosmic expansion, so dark energy acts in this sense as antigravity or cosmic repulsion. This can however occur within general relativity for substances with strongly negative pressure (tension); the equation of state, or pressure to energy density, ratio w measures this and when $w < -1/3$ then the substance acts in a gravitationally repulsive manner. Probing dark energy is regarded with such import because dark energy in turn probes the foundations of physics- either the nature of the quantum vacuum, the nature of gravity and spacetime, or their unification. By dominating the cosmic energy density dark energy determines the fate of the universe; continued acceleration would lead to an ever less dense and colder universe, with the horizon of the visible universe closing in around each observer, leaving observers in a truly dark universe. Dark energy is not detected tangibly because it smoothly permeates the entire universe. Indeed, in the case of a gravitational origin there may be no "thing" to detect; dark energy is manifest only as a change in physical laws, e.g. the inverse square law of gravitational attraction on certain length scales. Instead astronomers observe its direct effects accelerating the cosmic expansion and its indirect effects through the consequences of the acceleration on the contents of the universe. Measuring distances as a function of expansion scale factor a maps out the cosmic expansion history $a(t)$ and quantifies the deceleration and acceleration. The use of Type Ia supernovae as standardized light sources-calibrated candles-led to the observational discovery of dark energy by two groups in 1998 (Riess et al. 1998, Perlmutter et al. 1999).

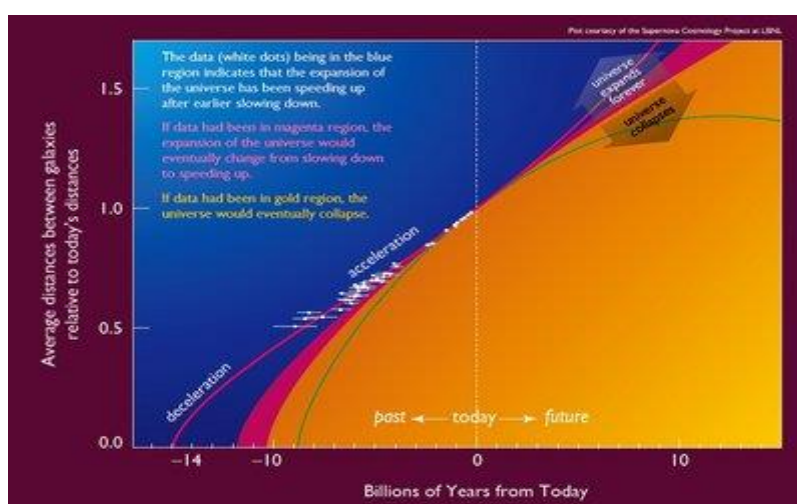


Figure 4. Observers view of the accelerating universe. The apparent brightness of supernovae gives a measure of the distance away and time taken for the light to reach us (horizontal axis), while the redshift of their spectra measures the expansion factor or change in average distance between galaxies or any points in space (vertical axis). Points shown in white are supernovae data that led to the 1998 discovery of the accelerating universe: they clearly lie on a curve in the blue region that requires a recent period of acceleration [9].

The path to understanding dark energy begins with a single question: has it always been the same throughout the history of the Universe? Unlike the search for dark matter (this quest is a young one. Scientists have known since the late 1920s that the

Universe is expanding, but it was assumed that expansion must be slowing down, as the force of gravity between galaxies and other matter puts on the brakes. In 1998, two teams found quite the opposite. They had been searching for stellar explosions of a particular type—type Ia supernovae, which occur when white-dwarf stars undergo a runaway nuclear reaction. The intrinsic brightness of a type Ia is fixed by how fast its light fades—brighter ones burn more briefly. So by counting how many days a type Ia takes to fade, you can work out how much light the explosion emitted; then, by measuring its apparent brightness at Earth, you can calculate how far away the supernova really is and how long the light has been travelling. This type of cosmological probe is called a standard candle. Measurements as of 2008, with the greatest weight coming from the combination of supernovae with either cosmic microwave background or baryon acoustic oscillation data, show that dark energy makes up $72 \pm 3\%$ of the total energy density of the universe, and its equation of state averaged over the last 7 billion years is $w = -1.00 \pm 0.1$ (Kowalski et al. 2008). This is consistent with the simplest picture, the cosmological constant, but also with a great many scenarios of time varying dark energy or extended gravity theories [10].

2.2. Baryon Acoustic Oscillations

So far, the most effective tool is based on cosmic sound waves. Shortly after the Big Bang, the Universe was filled with an elastic mixture of ions, electrons and radiation. Small density anomalies (created by quantum fluctuations in the first 10^{-32} seconds of the life of the Universe) gave this cosmic bell a tap, sending sound waves rippling outwards. After about 400,000 years, the Universe had cooled enough for ions to capture loose electrons. Because the resulting neutral atoms were transparent to radiation, letting photons whizz by them, the mixture was no longer elastic. And because sound needs an elastic medium to travel, the primordial sound waves were halted, imprinting an indelible pattern on the large-scale structure of the Universe. So instead of being positioned entirely at random, galaxies have a light tendency to be spaced at regular intervals. The characteristic distance has been growing as the Universe expands, and stands at about 500 million light years (153 mega parsec) today. Just as supernovae work as standard candles, these baryon acoustic oscillations (BAOs) can act as standard rulers. Mark the position of enough galaxies and you can measure the apparent size of BAOs. Compare that with the size predicted by their redshift, and you can work out how far away these particular BAOs are. By measuring the redshift of these galaxies, and plotting that against distance, it is possible to reveal how the expansion of space has behaved through cosmic history. The best view yet of BAOs was revealed in July by a Sloan Digital Sky Survey program called the Baryon Oscillation Spectroscopic Survey (BOSS). This is the largest such galaxy survey yet. “This technique is really coming into its own,” says Saul Perlmutter, a physicist at the University of California, Berkeley, who led one of the teams that discovered dark energy in 1998 and who received a share of the 2011 Nobel Prize in Physics along with Adam Riess and Brian Schmidt for the work. As well as backing up the supernova results within dependent evidence that expansion is accelerating, the BOSS data give some clues about how dark energy behaves. And the pattern of acceleration suggests that if dark energy is changing, it is not changing very fast. “The constant makes theory seem bizarrely asymmetric.” For now, that’s a conclusion that seems to favor a candidate for dark energy known as the cosmological constant [11].

2.3. Cosmological Constant

Some astronomers identify dark energy with Einstein's Cosmological Constant. In the context of dark energy, the cosmological constant is a reservoir which stores energy. Its energy scales as the Universe expands. Applied to the supernova data, it would distinguish effects due to the matter in the Universe from those due to the dark energy. Another explanation for how space acquires energy comes from the quantum theory of matter. In this theory, "empty space" is actually full of temporary ("virtual") particles that continually form and then disappear. But when physicists tried to calculate how much energy this would give empty space, the answer came out wrong—wrong by a lot. The number came out 10^{120} times too big. That's a 1 with 120 zeros after it. The cosmological constant is estimated by cosmologists to be on the order of 10^{-29}g/cm^3 , or about 10^{-120} in reduced Planck units. Particle physics predicts a natural value of 1 in reduced Planck units, leading to a large discrepancy. It's hard to get an answer that bad. The question is why the vacuum of space should have energy at all. Quantum-field theory posits a profusion of virtual particles that briefly come into existence and then disappear—a seemingly outrageous idea, but one that has allowed quantum theorists to make extremely accurate predictions of how ordinary particles interact. These virtual particles could be behind dark energy's repulsive force. But it's hard to make the numbers stack up. The vacuum energy needed to produce the observed cosmic acceleration is about 1 joule per cubic kilometer of space; the simplest version of quantum-field theory adds up the energy of those virtual particles to give a value about 120 orders of magnitude higher than that. Such dense vacuum energy would rapidly rip the Universe to shreds, and plainly that has not happened. Perhaps scientists are missing something. As-yet-undiscovered particles could cancel out the energy supplied by known particles. But, although it is simple to devise a theory that makes the value zero, it is hard to almost -but- not-exactly cancel out a huge number to leave the small required value of vacuum energy. "The cosmological constant is an odd beast," says Perlmutter. "It makes the theory seem bizarrely asymmetric." More recently, the WMAP seven-year analysis gave an estimate of 72.8% dark energy, 22.7% dark matter and 4.6% ordinary matter [12]. The cosmological constant has negative pressure equal to its energy density and so causes the expansion of the universe to accelerate. The reason why a cosmological constant has negative pressure can be seen from classical thermodynamics; energy must be lost from inside a container to do work on the container. A change in volume dV requires work done equal to a change of energy $-PdV$, where P is the pressure. But the amount of energy in a container full of vacuum actually increases when the volume increases (dV is positive), because the energy is equal to ρV , where ρ (rho) is the energy density of the cosmological constant. Therefore, P is negative and, in fact, $P=-\rho$.

2.4. Quintessence

Quintessence means fifth essence; remember Earth, Water, Fire, and Air, the 'four essences' of the Ancient Greeks. Well, in modern cosmology, there are also four essences: normal matter, radiation (photons), cold dark matter, and neutrinos (which are hot dark matter!). Quintessence covers a range of hypotheses (or models); the main difference between quintessence as a (possible) explanation for dark energy and the cosmological constant Λ (which harks back to Einstein and the early years of the 20th century) is that quintessence varies with time (albeit slowly), and can also vary with location (space). One version of quintessence is phantom energy, in which the energy density increases with time, and leads to a Big Rip end of the universe. Quintessence is a hypothetical form of dark energy, more precisely a scalar field, postulated as an explanation of the observation of an accelerating rate of expansion of

the universe, rather than due to a true cosmological constant. Quintessence was the first cab off the rank as an alternative. It's postulated to be an energy field, like the Higgs field, which gives particles mass. Unlike the Higgs field, which has no energy density in the Universe, a quintessence field could have an energy density that would change overtime, but not be zero. Einstein's cosmological constant always stays the same, so if the rate of expansion of the Universe doesn't reflect a constant energy for space, that would indicate the existence of something like quintessence. The problem is, a reasonable quintessence field may change by only 1 part in 10,000 over the age of the Universe, and we would only be able to measure it to 1 part in 10, at best. A prominent example is provided by a time-dependent homogeneous minimally coupled scalar field $\phi(t)$ called Quintessence. It is well known that such a field can be viewed as a comoving perfect fluid with

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (1)$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (2)$$

Hence the corresponding equation of state parameter is

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \quad (3)$$

We see that w_ϕ can be sufficiently negative in this case provided the kinetic energy is small enough compared to the potential energy. This mechanism is actually used in implementing the inflationary scenario in the very early universe. Though w_ϕ is generically dynamical, it can never cross the boundary $w=-1$. Hence a time-varying equation of state parameter is a natural outcome for quintessence models with the important restriction that there is no phantom phase [13]. Can quintessence be observed; or, rather, can quintessence be distinguished from a cosmological constant? In astronomy, yes...by finding a way to observed (and measure) the acceleration of the universe at widely different times (quintessence and Λ predict different results). Another way might be to observe variations in the fundamental constants (e.g. the fine structure constant) or violations of Einstein's equivalence principle. One project seeking to measure the acceleration of the universe more accurately was ESSENCE ("Equation of State: SupEr Novae trace Cosmic Expansion") [14].

2.5. $f(R)$ Gravity

$f(R)$ gravity is a class of effective theories representing a new approach to the gravitational interaction. The paradigm is that Einstein's General Relativity has to be extended in order to address several shortcomings emerging at ultraviolet and infrared scales. These are essentially due to the lack of a final, self-consistent theory of quantum gravity. From the astrophysical and cosmological viewpoints, the goal is to encompass phenomena like dark energy and dark matter under a geometric standard related to the possibility that gravitational interaction depends on the scales. This *geometric* view, in principle, does not need the introduction of further particle ingredients and preserves all the well-posed results of General Relativity, being based on the same fundamental principles (Equivalence Principle, diffeomorphism

invariance, gauge invariance, etc.). The main criticism to this approach is that, until now, no $f(R)$ model, or any Extended Theory of Gravity, succeeds in addressing the whole phenomenology ranging from quantum to cosmological scales. Besides, the $f(R)$ description of dark side of the universe is substantially equivalent to that related to the hypothesis of dark material constituents. The need of one or more than one *experiment* capable of retaining or ruling out one of the two concurring pictures, is pressing to solve the debate [15].

3. Modification of the Laws of Gravity Rules out Dark Energy

3.1. Criticisms

The 1990's two teams of astronomers, the Supernova Cosmology Project (Lawrence Berkeley National Laboratory) and the High-Z Supernova Search (international) were looking for distant Type Ia supernovae in order to measure the expansion rate of the Universe with time. They expected that the expansion would be slowing, which would be indicated by the supernovae being brighter than their redshifts would indicate. Instead, they found the supernovae to be fainter than expected from a uniformly expanding universe. Hence, the expansion of the Universe was accelerating! [17]. In the flat Universe where the curvature is zero and the density is the critical density, new hypothetical objects, dark energy and dark matter are essential to bridge the gap between theory and observation. It is clear that these new ingredients, dark energy and dark matter, arose as a consequence to the false flat universe paradigm. Without them, there would be a fatal contradiction between the observations made by astronomers and the predictions of the Big Bang theory. Current models of the universe require dark energy to explain the observed acceleration in the rate at which the universe is expanding. Scientists base this conclusion on measurements of the distances to Type Ia supernovae in distant galaxies, which appear to be farther away than they would be if the universe's expansion was not accelerating. On the even larger cosmic scales of an expanding universe, gravity appears to be weaker than expected in a universe containing only normal matter and dark matter. Scientists invented a new force, called "dark energy", a sort of anti-gravitational force causing acceleration in the expansion of the universe out from the big bang 13.8 billion years ago. The dominant force in cosmology is gravity. Einstein's theory of General Relativity, our modern theory of gravity, forms a solid theoretical underpinning of the recent progress. On the experimental side the discovery by Hubble of the expanding universe is the basis of the Big Bang model of a dynamic universe that allows a meaning full discussion of a beginning, and thus an age of the universe. The discovery of the Cosmic Microwave Background (CMB), the relic sea of photons left over from the early universe, further supports the Big Bang model. The realization from the rotation curves of galaxies that there is considerably more matter with attractive gravity, called dark matter, than we can see via electromagnetic radiation (the visible matter) complicates things. With attractive gravity between the components of the universe the expectation was that the expansion velocity of the universe, the so called Hubble velocity, would decrease with time, i.e. a decelerating universe. The recent discovery that the Hubble velocity is increasing with time, an accelerating universe, was an immense surprise. This led to the postulation of the existence of a new component of the universe, dark energy, with some very unusual properties, not the least among which is that it has, in some sense of the word, repulsive gravity that drives the acceleration. The best estimates that fit all of the presently existing data is that the energy/matter density of the universe is made of, 4% of visible matter, dark matter making up 24% and dark energy making

up the majority 72%. In particular there is a focus on the measurement of the equation of state parameter w , which can distinguish between a cosmological constant ($w=-1$) or some other form of dark energy with $w < -1/3$ but not exactly -1 . The present dependence of w , which would be inconsistent with a cosmological constant, is very poorly constrained by the present data. Its results, $w = -0.93 \pm 0.16$ is consistent with the cosmological constant but higher precision may yet show a deviation from -1 . The parameter w_a , indicating a time interesting to point out that if it turns out that dark energy is due to a nonzero cosmological constant, a value of Ω_Λ in the vicinity of 0.7 would be implied. Such a small value of Ω_Λ would lead to one of the most dramatic inconsistencies in modern physics called the cosmological constant problem [18]. Modern field theory of particle physics predicts that the energy density of the vacuum contributes to the cosmological constant with a value 120 orders of magnitude larger than the experimentally observed value. Actually this problem would persist even if the dark energy turned out to be something else than the cosmological constant, with $\Lambda=0$. Even though many theorists have tried, there is no explanation insight to this fundamental disagreement. Accelerating is ruling out a nonaccelerating universe and a zero cosmological constant with a very high level of confidence. All of the surveys are consistent with $w=-1$, assuming a constant w and a flat universe. *We exhibit the hyperbolic structure of the universe that explains the accelerating expansion of the universe without need for an additional components, dark energy and dark matter.* Most regions of spacetime are flat over only a limited range of space and time. Evidence that a frame is not inertial (so that its region of spacetime is not flat) is the relative acceleration ("tidal acceleration") of a pair of free test particles with respect to one another. If tidal accelerations affect an experiment in a region of spacetime, then we say that spacetime region is curved, and special relativity cannot validly be used to describe this experiment. In that case we must use General Relativity, the theory of gravitation, which correctly describes the relations among events spread over regions of spacetime too large for Special Relativity [19]. The core principles underlying standard cosmology are, "Physics is the same everywhere in the universe, but without this assumption of universality we could not undertake a scientific study of the universe in the large. The local physics that applies everywhere determines the dynamics of the universe in the large. That is, there is not a 'cosmological physics' that applies only on large scales; rather large scale dynamics emerges in a bottom-up way from the combined effects of local dynamics on matter everywhere on small scales." However major unknowns remain, in particular the nature of dark matter and dark energy". [20] The previous postulates seem absurd. Why the *universality* of the laws of physics break down when applied to predict Mercury's orbit, while such laws fit the rest of the Solar system? Our main mistake is that we accept an unverifiable assumption that the portion of the universe which can be observed is representative of the whole, and that the laws of physics are the same throughout the entire universe. It is an oversimplification to generalize that the universe is globally flat and the laws of physics are the same throughout the whole universe. Newton physics is not relativistic; it is only valid locally through a flat space. The physics of the large structure must be relativistic. Newton's laws of gravity do no longer hold in non-Euclidean geometry. Geometry is the study of the local structures of the manifold, by means of measurement and observations. Observations being made are not complete in themselves; they interpreted within a theory (a paradigm). Topology is the study of the global structures of the manifold, mathematically. In General Relativity Theory, gravity is geometry. In order to modify the laws of gravity; first we should modify the underpinning geometry itself. Hence, we establish the modified laws of gravity from

the updated non-Euclidean geometry. We prove that the geometry of the universe is globally hyperbolic. We develop the laws of gravity in the hyperbolic spacetime. Such modified laws fit the current observed data and successively predict the flat rotation curve and the accelerated expansion of the universe without invoking Dark Matter and Dark Energy. The observed average density of the universe is $3 \times 10^{-31} \text{g/cm}^3$. This density implies an open universe (hyperbolic universe), contradicts the Inflation Theory which predicts a flat universe with a hypothetical critical density of 10^{29}gm/cm^3 [21]. What density must we accept; the observed or the hypothetical? The hypothetical critical density necessary for a flat universe requires mysterious ingredients: Dark Matter and Dark Energy that have never been observed. The hyperbolic universe consists of zero cosmological constant [22]. A flat-dust universe with zero pressure models is an oversimplification assumed to solve Einstein's Field equations. This oversimplification led to a drastic failure in predicting the missing mass and the accelerating expansion of the universe. Assuming a flat universe, astronomers expected that the expansion of the universe would be slowing, which would be indicated by the supernovae being brighter than their redshifts would indicate. Instead, they found the supernovae to be fainter than expected from a uniformly expanding universe. The motion on straight line (flat space) is uniform (not accelerated). The motion through a curved path is accelerated. So, if the universe is not flat as assumed, the accelerated expansion is not due a dark energy; it's an attribute of the curved spacetime (Hyperbolic spacetime and open universe).

Dark energy isn't noticeable on small scales but becomes the dominating force of the universe on the largest cosmic scales (why?). Locally the spacetime is flat, there is no cosmological redshift. For example, observations tell us that space within galaxies, which are rather diffuse objects, do not expand. Thus, where is the "border line" in space which divides expanding space from nonexpanding space? Two galaxies within our Local Group, including Andromeda, and a few galaxies in the Virgo Cluster display blueshifts and so are moving toward us, but these results from their local motion (peculiar velocity). Why nearby galaxies exhibit blue-shift? Because their peculiar velocities are greater than their recession velocities! The answer is more convenient if we say: Locally the spacetime is flat through which the curvature is negligible (no cosmological redshift), where the random peculiar velocity dominates. If cosmological redshift has nothing to do with the Doppler Effect, how do we know that galaxies that are very far away are also receding from us? How to compare between two unrelated concepts, the Doppler redshift and the cosmological redshift? Andromeda galaxy is blueshifted because it's sufficiently nearby where the space time is approximately flat and Special Relativity works. It's blue shifted according to the Doppler Effect in flat spacetime [23]. The observed redshift doesn't exhibit an accelerated expansion within the spacetime of galaxy and a cluster of galaxies (no matter how large their spacetime), while it does exhibit such accelerated expansion throughout the entire cosmos. The observed redshift might be interpreted due to a different cause. According to General Relativity gravity is not a force (as in Newton physics), it's the curvature of the spacetime. Einstein's equivalence principle states: It's impossible to detect the gravitational field (the curvature) locally (on small scales). The curvature accounts for the supernovae to be fainter than expected from a uniformly expanding universe. The curvature dominates the large structure, hence it's the cause of the appeared accelerated expansion of the universe.

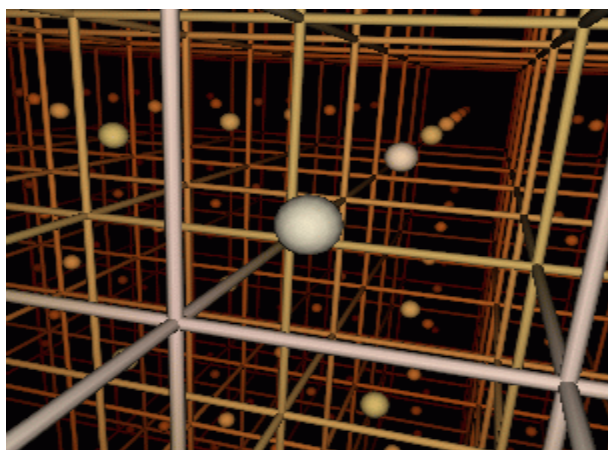


Figure 5. In the Euclidean geometry space is divided into cubes and one experiences the ordinary, familiar perspective: the apparent angular size of objects proportional to the inverse of their distance [24].

"We are confronted with the need to reconsider the guiding principles that have been used for decades to address the most fundamental questions about the physical world. These are symptoms of a phase of crisis. And yet, this superb monument of knowledge is insufficient to address some fundamental questions. The Standard Model is incapable of shedding light on the dynamics underlying electroweak symmetry breaking or explaining the structure of quarks, leptons, and their mass pattern at a fundamental level. The theory of inflation, in spite of its stunning conceptual successes, could not be linked univocally with a unified theory of particle physics. Moreover, the ubiquitous phenomenon of eternal inflation has changed the perspective on the outcome of an inflationary universe and its properties. We have plausible explanations for the cosmic baryon asymmetry, but we lack any conclusive empirical confirmation. The nature of dark matter is still unknown. The observed value of the cosmological constant is hard to reconcile with the rules of effective field theory, and quantum gravity is still beyond our grasp. None of these problems are new, and theoreticians have been tackling them for decades. What is changing is the feeling that the paradigm that so successfully led to the Standard Model may not be the right tool to make further progress. There is a wide spread sensation that the organising principles based on symmetry and separation of scales, which follow from an effective quantum field theory approach, in spite of their triumphs, must be superseded by new organising principles. Physicists are in search for new conceptual paradigms, which is another symptom of a phase of crisis. Crisis means the opportunity to look at a problem with new eyes; it is a moment of change, a discontinuity between past and future. Crisis does not mean a decline of ideas, but the search for a paradigm change" [25].

3.2. Conflicts in the Observing Dark Energy Projects

The observations in measuring the Dark Matter, Dark Energy and the local flatness are a mess. Discrepancy between observations could point to new physics.

COBE (Cosmic Background Explorer): Confirmed that the universe is flat to within 10%. The maps were recorded by the Boomerang experiment, a collaboration between astrophysicists from Italy, Canada, the UK and the US [33].

Planck satellite: Space within our cosmic horizon is curved by not more than 1%. Hubble constant is 67.15; Planck's Dark Matter is 29%. One of the most surprising

findings is that the fluctuations in the CMB temperatures at large angular scales do not match those predicted by the standard model – their signals are not as strong as expected from the smaller scale structure revealed by Planck.

Dark Energy Survey (DES): DES released data on 3 August 2018 suggest that the clumping has happened more slowly than indicated by earlier estimates, The Kilo Degree Survey (KiDS) also found a similar discrepancy last year. The researchers find Dark Energy 73% and dark matter about 26%. WMAP's dark matter is 23%! The level of concentration (the lumpiness of the Universe) measured by DES is 7% lower than what the standard model of cosmology predicts, based on Planck's data from the primordial Universe. DES, find the speed of expansion to be about 8% faster than that predicted based on Planck data. If both the new measurement of the Hubble constant and the earlier measurements by the Planck team are accurate, then something in the standard model has to *change*, Riess says [32]. Universe is expanding. One camp of scientists, the same camp that won the Nobel Prize for discovering dark energy, measured the expansion rate to be 73 km/s/Mpc, with an uncertainty of only 2.4%. But a second method, based on the leftover relics from the Big Bang, reveals an answer that's incompatibly lower at 67 km/s/Mpc, with an uncertainty of only 1%. If everyone measured the same rate for the expanding Universe, there would be nothing to challenge this picture, known as Λ CDM. But everyone doesn't measure the same rate. Currently, the fact that distance ladder measurements say the Universe expands 9% faster than the leftover relic method is one of the greatest puzzles in modern cosmology. A mathematical discrepancy in the expansion rate of the Universe is now "pretty serious", and could point the way to a major discovery in physics, says a Nobel laureate; Prof Riess. The observer is operating within a "paradigm". Observations being made are not complete in themselves; they interpreted within a theory (a paradigm). Moreover observations are based on assumptions. The accuracy of the observations relies on the efficiency of the assumptions. The observations of Dark Matter, Dark Energy and Inflation pre-assumed a flat universe. If we interpreted the previous observations within the General Relativity, the apparent discrepancy would be removed. Since in general Relativity the spacetime locally is flat and globally is curved (hyperbolic).

3.3. The Hyperbolic Geometry of the Universe

We solved [26] the dynamical equations of cosmology (*Friedmann's equations*)

$$\dot{R}^2 + k = (8\pi / 3)\rho R^2 \quad (4)$$

$$2R\ddot{R} + \dot{R}^2 + k = -8\pi p R^2 \quad (5)$$

Where $\mathbf{p}$ is the pressure and ρ is the energy density of the cosmological fluid and $\mathbf{k}$ is the curvature. The solution of Eq.(4), is

$$\begin{aligned} R &= -i\sqrt{3k / 8\pi\rho_j} \sinh t\sqrt{8\pi\rho_j / 3} \\ k &= -1 \\ R &= \sqrt{3 / 8\pi\rho_j} \sinh t\sqrt{8\pi\rho_j / 3} \end{aligned} \quad (6)$$

Note that the solution represented by Eq.(6) is evaluated only for the values simultaneously associated with ρ_j , namely $\mathbf{R}_j, t_j$

$$R_j = \sqrt{3 / 8\pi\rho_j} \sinh t_j \sqrt{8\pi\rho_j / 3} \quad (7)$$

(ii)-We shall see that the solution of equation (4) satisfies the second order differential equation (5) in order to be consistent.

We have from the solution of Eq.(4) for any chosen fixed value ρ_j

$$R = \sqrt{\frac{3}{8\pi\rho_j}} \sinh t \sqrt{\frac{8\pi\rho_j}{3}} \quad (8)$$

$$\dot{R} = \cosh t \sqrt{\frac{8\pi\rho_j}{3}} \quad (9)$$

$$\ddot{R} = \sqrt{\frac{8\pi\rho_j}{3}} \sinh t \sqrt{\frac{8\pi\rho_j}{3}} = \frac{8\pi\rho_j}{3} R \quad (10)$$

Substitute these values in Eq.(2), and put $k=-1$, yields:

$$\begin{aligned} 2R\ddot{R} + \dot{R}^2 - 1 &= -8\pi p R^2 \\ 2R\left(\frac{8\pi\rho_j}{3} R\right) + \cosh^2 \sqrt{\frac{8\pi\rho_j}{3}} - 1 &= -8\pi p R^2 \\ 2R^2\left(\frac{8\pi\rho_j}{3}\right) + \sinh^2 \sqrt{\frac{8\pi\rho_j}{3}} &= -8\pi p R^2 \\ 2R^2\left(\frac{8\pi\rho_j}{3}\right) + \frac{8\pi\rho_j}{3} R^2 &= -8\pi p R^2 \\ 8\pi\rho_j R^2 &= -8\pi p R^2 \\ p &= -\rho_j \end{aligned} \quad (11)$$

The last equation is known as the equation of state of cosmology. The argument of the solution predicts the equation of state of cosmology $p = -\rho_j$. Since the energy density is always positive, the negative pressure implies an accelerated expansion of the universe. Hence equations (1) and (2) are consistent for any chosen fixed value ρ_j of the parameter ρ . *The argument of the solution predicts the equation of state $p = -\rho_j$.*

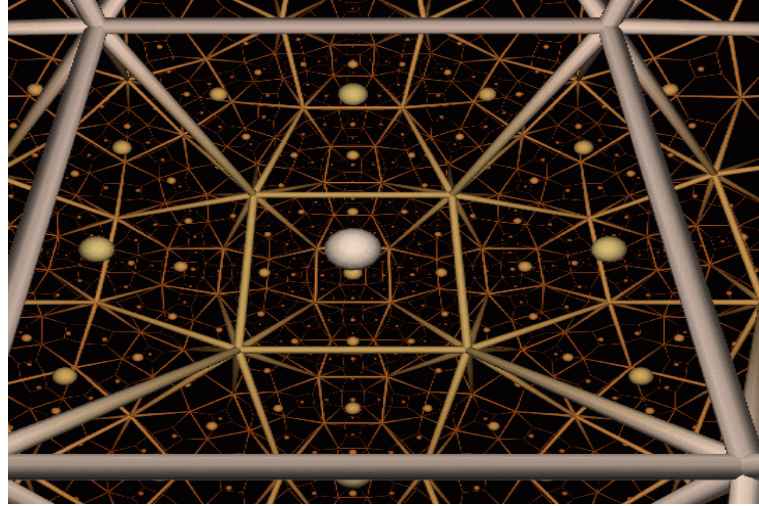


Figure 6. Hyperbolic space shown here is tiled with regular dodecahedra. In Euclidean space such a regular tiling is impossible. The size of the cells is of the same order as the curvature scale. Although perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space, the apparent angular size of distant objects falls off much more rapidly, in fact exponentially, as can be seen in the figure [27].

3.4. Method of Solution of the Dynamical Equations in the Presence of the Cosmological Constant

(i) The empty spacetime case:

The dynamical equations of cosmology for the empty space-time in the presence of cosmological constant are given by:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} = 0 \quad (12)$$

$$\therefore \dot{R}^2 + k - \Lambda R^2/3 = 0 \quad (13)$$

$$2R\ddot{R} + \dot{R}^2 + k - \Lambda R^2 = 0 \quad (14)$$

$$\because \Lambda = 8\pi\rho^{\text{vacuum}} \quad (15)$$

$$\therefore \dot{R}^2 + k - 8\pi\rho^{\text{vacuum}}R^2/3 = 0 \quad (15)$$

$$2R\ddot{R} + \dot{R}^2 + k - 8\pi\rho^{\text{vacuum}}R^2 = 0 \quad (16)$$

The vacuum energy is constant since it is not dilute with time, hence

$$\dot{R}^2 + k = 8\pi\rho^{vacuum}R^2/3 \quad (17)$$

$$\int_0^R dr / \sqrt{r^2 - 3k / 8\pi\rho^{vacuum}} = \sqrt{8\pi\rho^{vacuum} / 3} \int_0^t d\tau \quad (18)$$

$$R = -i\sqrt{3k / 8\pi\rho^{vacuum}} \sinh t \sqrt{8\pi\rho^{vacuum} / 3} \quad (19)$$

$$\therefore k = -1$$

$$R = \sqrt{3 / 8\pi\rho^{vacuum}} \sinh t \sqrt{8\pi\rho^{vacuum} / 3} \quad (20)$$

Substitute equation (20) into equation (16) in order to be consistent, we get

$$R = \sqrt{3 / 8\pi\rho^{vacuum}} \sinh t \sqrt{8\pi\rho^{vacuum} / 3}$$

$$\dot{R} = \cosh t \sqrt{8\pi\rho^{vacuum} / 3}$$

$$\ddot{R} = \sqrt{8\pi\rho^{vacuum} / 3} \sinh t \sqrt{8\pi\rho^{vacuum} / 3}$$

$$= 8\pi\rho^{vacuum} / 3 R$$

$$2R\ddot{R} + \dot{R}^2 + k - \Lambda R^2 = 0$$

$$2R\ddot{R} + \dot{R}^2 - 1 - 8\pi\rho^{vacuum}R^2 = 0$$

$$2R^2 \cdot 8\pi\rho^{vacuum} / 3 + \cosh^2 t \sqrt{8\pi\rho^{vacuum} / 3} - 1 - 8\pi\rho^{vacuum}R^2 = 0$$

$$2R^2 \cdot 8\pi\rho^{vacuum} / 3 + \sinh^2 t \sqrt{8\pi\rho^{vacuum} / 3} - 8\pi\rho^{vacuum}R^2 = 0$$

$$2R^2 \cdot 8\pi\rho^{vacuum} / 3 + 8\pi\rho^{vacuum} / 3 R^2 - 8\pi\rho^{vacuum}R^2 = 0$$

$$8\pi\rho^{vacuum}R^2 - 8\pi\rho^{vacuum}R^2 = 0$$

$$0 = 0$$

(21)

(ii) The non empty spacetime case:

The dynamical equations of cosmology for the non empty space-time in the presence of cosmological constant are given by:

$$\begin{aligned}\dot{R}^2 + k - \Lambda R^2 / 3 &= (8\pi / 3) \rho R^2 \\ 2R\ddot{R} + \dot{R}^2 + k - \Lambda R^2 &= -8\pi p R^2 \\ \therefore \Lambda &= 8\pi \rho^{\text{vacuum}} \\ \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 / 3 &= (8\pi / 3) \rho R^2\end{aligned}\quad (22)$$

$$2R\ddot{R} + \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 = -8\pi p R^2 \quad (23)$$

Since the cosmological constant represents about 74% of the matter of the universe, it dominates $\rho^{\text{vacuum}} + \rho_j$. So it is plausible that $\rho^{\text{vacuum}} + \rho_j$ is approximately constant, hence

$$\dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 / 3 = (8\pi / 3) \rho_j R^2 \quad (24)$$

$$2R\ddot{R} + \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 = -8\pi p R^2 \quad (25)$$

$$R = \sqrt{3 / 8\pi \rho^{\text{vacuum}} + \rho_j} \sinh t \sqrt{8\pi \rho^{\text{vacuum}} + \rho_j} / 3 \quad (26)$$

Substitute Eq.(11) in Eq.(10) gives

$$\begin{aligned}2R\ddot{R} + \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ 2R\ddot{R} + \dot{R}^2 - 1 - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ 8\pi \rho^{\text{vacuum}} + \rho_j R^2 - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ \rho^{\text{vacuum}} + \rho_j - \rho^{\text{vacuum}} &= -p \\ \therefore p &= -\rho_j\end{aligned}\quad (27)$$

3.5. Hyperbolic Universes Does Not Need Dark Energy

Although perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space (i.e. the Universe locally is approximately flat consistent with local observations), the apparent angular size of distant objects falls off much more rapidly, in fact exponentially. The Universe is globally hyperbolic as we did prove mathematically. Such a solution predicts the equation of state of cosmology, $p = -\rho$. The hyperbolic structure of the space causes the accelerated expansion of the universe equivalent to its negative pressure. The current observed density of the universe is $\rho = 10^{-31} \text{g/cm}^3$, consistent with a hyperbolic open universe.

Incorrect zero pressure-dust universe model: Einstein postulates [31] that the matter dominated universe could be modeled as dust with zero pressure in order to simplify and solves Friedmann's equations.

$$\begin{aligned}\dot{R}^2 + k &= (8\pi / 3)\rho R^2 \\ 2R\ddot{R} + \dot{R}^2 + k &= 0 \\ 2\ddot{R}/R + (8\pi / 3)\rho &= 0 \\ \ddot{R} &= -(8\pi / 3)\rho R/2 < 0 \\ \therefore \ddot{R} &< 0\end{aligned}$$

The pressure less form of Eq.(5) describes a decelerating expansion state of the universe which is described by the energy tensor of matter for dust where $p=0$. We solved the second dynamical equation of cosmology, the space-space component; in it is pressure less form:

$$\begin{aligned}2R\ddot{R} + \dot{R}^2 + k &= 0 \\ \therefore k &= -1 \\ 2R\ddot{R} + \dot{R}^2 - 1 &= 0\end{aligned}$$

$R = t$, satisfies the last differential equation. Substitute $t = R, k = -1$ in the first dynamical equation Eq.(4)

$$\begin{aligned}\dot{R}^2 + k &= (8\pi / 3)\rho R^2 \\ (1)^2 - 1 &= (8\pi / 3)\rho R^2 \\ 0 &= (8\pi / 3)\rho R^2 \\ \therefore \rho &= 0\end{aligned}\tag{28}$$

Hence the zero pressure does not lead to a dusty universe. In fact zero pressure Universe is an empty space, since $\rho = 0$.

In the presence of pressure, from Eq.(4) and (5) we can obtain.

$$\begin{aligned}\ddot{R} &= -\frac{4\pi}{3} \rho + 3p/R \\ \therefore p &= -\rho \\ \therefore \ddot{R} &= \frac{8\pi}{3} R > 0\end{aligned}$$

The positive acceleration guarantees an accelerating expansion of the universe.

Newton first law states that the body keeps moving with a uniform velocity in straight line. Similarly, the free fall of an object in a flat spacetime is uniform. An

accelerated motion is described by a curve. The distant objects -e.g. supernovae-were influenced under the curvature of the spacetime. They possess an accelerating free fall due to the curvature of the hyperbolic spacetime that manifests itself by the equation of the state $p = -\rho$ which is the property of the hyperbolic structure of the Universe. The universe is not flat. We did prove that, the universe globally is hyperbolic. The hyperbolic universe doesn't need dark energy to account for the accelerating expansion. The equation of the state $p = -\rho$ associated with the hyperbolic universe, derives such an accelerated expansion.

The accelerating expansion is a property of the Hyperbolic Universe: One explanation for dark energy is that it is a property of space. The simplest explanation for dark energy is that it is simply the "cost of having space": that is, a volume of space has some intrinsic, fundamental energy. Some astronomers identify dark energy with Einstein's Cosmological Constant. In the context of dark energy, the cosmological constant is a reservoir which stores energy. Its energy scales as the Universe expands. Applied to the supernova data, it would distinguish effects due to the matter in the Universe from those due to the dark energy. Another explanation for how space acquires energy comes from the quantum theory of matter. In this theory, "empty space" is actually full of temporary ("virtual") particles that continually form and then disappear. But when physicists tried to calculate how much energy this would give empty space, the answer came out wrong-wrong by a lot. The number came out 10^{120} times too big. That's a 1 with 120 zeros after it. The cosmological constant is estimated by cosmologists to be on the order of 10^{-29}g/cm^3 , or about 10^{-120} in reduced Planck units. Particle physics predicts a natural value of 1 in reduced Planck units, leading to a large discrepancy. It's hard to get an answer that bad.

Note that, Friedmann's equation in the presence of the cosmological constant are given by

$$\dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 / 3 = (8\pi / 3) \rho_j R^2 \quad (29)$$

$$2R\ddot{R} + \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 = -8\pi p R^2 \quad (30)$$

$$R = \sqrt{3 / 8\pi \rho^{\text{vacuum}} + \rho_j} \sinh t \sqrt{8\pi \rho^{\text{vacuum}} + \rho_j} / 3 \quad (31)$$

Substitute Eq.(31) in Eq.(30) and $k=-1$ gives

$$\begin{aligned} 2R\ddot{R} + \dot{R}^2 + k - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ 2R\ddot{R} + \dot{R}^2 - 1 - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ 8\pi \rho^{\text{vacuum}} + \rho_j R^2 - 8\pi \rho^{\text{vacuum}} R^2 &= -8\pi p R^2 \\ \rho^{\text{vacuum}} + \rho_j - \rho^{\text{vacuum}} &= -p \\ \therefore p &= -\rho_j \end{aligned} \quad (32)$$

So the cosmological constant (the vacuum energy) disappeared in the solution of the second differential Eq.(30). Just the ordinary energy density state ρ_j remains in the Hyperbolic Universe to derive the accelerating expansion equivalent to its negative pressure. Hyperbolic Universe involves zero [22] cosmological constant (the vacuum energy). The negative pressure $p = -\rho_j$ is the property of the hyperbolic structure of the Universe. The hyperbolic structure of the spacetime –not the cosmological constant-causes the apparently observed accelerating expansion of the Universe. Moreover the equation of the hyperbolic time evolution of the Universe predicts the observed current large structure of the observable Universe 10^{28} cm associated with 14×10^9 yr without invoking Dark Energy, as follows: The Hyperbolic Universe expand, through the hyperbolic time evolution equation (7), legitimately to 10^{28} cm, very consistent with the current observable universe [28].

Note that in geometrical units:

$$1\text{sec}=2.997 \times 10^{10}\text{cm}$$

$$1\text{gram}=7.425 \times 10^{-29}\text{cm}$$

$$1\text{yr}=3.16 \times 10^7\text{s}$$

$$\text{The energy density now } \rho_{\text{now}} = 10^{-31}\text{g/cm}^3 = 7.425 \times 10^{-60}\text{cm}^{-2}$$

$$\text{The age of the Universe (approximately) } t_{\text{now}} = 14 \times 10^9\text{yr}$$

Substitute the above data in the hyperbolic time evolution equation of the Universe, yields

$$\begin{aligned} R_j &= \sqrt{3/8\pi\rho_j} \sinh\left[t_j \sqrt{8\pi\rho_j/3}\right] \\ R_{\text{now}} &= \sqrt{3/8\pi\rho_{\text{now}}} \sinh\left[t_{\text{now}} \sqrt{8\pi\rho_{\text{now}}/3}\right] \\ R_{\text{now}} &= \sqrt{3 / 8\pi \times 7.425 \times 10^{-60}} \times \\ &\sinh\left[\frac{14 \times 10^9 \times 3.16 \times 10^7 \times 2.997 \times 10^{10}}{\sqrt{8\pi \times 7.425 \times 10^{-60} / 3}}\right] \\ R_{\text{now}} &= 1.6 \times 10^{29} \times \sinh 0.08287 \\ R_{\text{now}} &= 1.3 \times 10^{28}\text{cm} \end{aligned} \tag{33}$$

Topic	Size	Mass/Energy	First Appearance
Macro-world			
Observable Universe ^a 	1.3×10^{28} cm. (cosmic horizon)	$4 \times 10^{22} M_{\text{sun}}$ (including dark energy, ordinary and dark matters)	0 sec.
Superclusters ^a 	10^{26} cm.	$10^{16} M_{\text{sun}}$	11×10^9 yr.
Clusters of Galaxies ^a 	10^{24} cm.	$10^{15} M_{\text{sun}}$	6×10^9 yr.
Galaxies ^a 	10^{22} cm.	$10^{11} - 10^{14} M_{\text{sun}}$	7×10^8 yr.
Star Clusters ^a 	10^{20} cm.	$10^2 - 10^6 M_{\text{sun}}$	5×10^8 yr.
Planetary Systems ^a 	10^{16} cm.	$0.1 - 100 M_{\text{sun}}$	1.8×10^8 yr.
Stars ^a 	10^{11} cm.	$0.1 - 100 M_{\text{sun}}$	1.8×10^8 yr.
Earth ^g 	10^9 cm.	6×10^{27} gm.	9.5×10^9 yr.

Figure 7. The Cosmic Table, the current observed universe 1.3×10^{28} cm. [29]

The Hyperbolic Universe grows exponentially preserve a legitimate inflation covers the current observed large structure. Hence, the horizon problem also does no longer exist, since the backward exponential contraction re-put both sides of the Universe at causal contact. Penrose said "If the curvature $k \neq 0$, then inflation is out". [30].

3.6. The Value Of Hubble Constant *n* The Hyperbolic

$$\begin{aligned}
 \rho_{\text{observed}} &= 3 \times 10^{-31} \text{ g/cm}^3 \\
 \therefore 1 \text{ g} &= 7.425 \times 10^{-29} \text{ cm} \\
 \therefore \rho_{\text{observed}} &= 3 \times 7.425 \times 10^{-60} \text{ cm}^{-2}
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 \dot{R}^2 + k &= \frac{8\pi}{3} \rho R^2 \\
 \rho_{\text{observed}} &= 3 \times 7.425 \times 10^{-60} \text{ cm}^{-2} \\
 H^2 &= \frac{\dot{R}^2}{R^2} = \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} \rho_{\text{observed}} - \frac{k}{R^2} \\
 \therefore k &= -1 \\
 H^2 &= \frac{8\pi \times 3 \times 7.425 \times 10^{-60}}{3} \text{ cm}^{-2} + \frac{1}{1.3 \times 10^{28}^2 \text{ cm}^2} \\
 &= \left[\frac{8\pi \times 3 \times 7.425}{3} + \frac{10^4}{1.69} \right] \times 10^{-60} \text{ cm}^{-2}
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 H &= \sqrt{\left[\frac{8\pi \times 3 \times 7.425}{3} + \frac{10^4}{1.69} \right]} \times 10^{-60} \text{ cm}^{-2} \\
 H &= \sqrt{\left[\frac{8\pi \times 3 \times 7.425}{3} + \frac{10^4}{1.69} \right]} \times 10^{-30} \text{ cm}^{-1} \\
 H &= \sqrt{186.6105 + 5917.1598} \times 10^{-30} \text{ cm}^{-1} \\
 H &= \sqrt{6103.77} \times 10^{-30} \text{ cm}^{-1} \\
 H &= 78.1266 \times 10^{-30} \text{ cm}^{-1}
 \end{aligned} \tag{36}$$

$$\begin{aligned}
 \text{Mpc} &= 3.09 \times 10^{25} \text{ cm} \\
 1 \text{ sec} &= 2.997 \times 10^{10} \text{ cm} \\
 1 \text{ km} &= 10^6 \text{ cm}
 \end{aligned} \tag{37}$$

$$\begin{aligned}
 \left[\text{km/sec} / \text{Mpc} \right] &= \left[10^6 \text{ cm} / 2.997 \times 10^{10} \text{ cm} / 3.09 \times 10^{25} \text{ cm} \right] \\
 \left[\text{km/sec} / \text{Mpc} \right] &= 1.08 \times 10^{-30} \text{ cm}^{-1} \\
 \text{cm}^{-1} &= \frac{\left[\text{km/sec} / \text{Mpc} \right]}{1.08 \times 10^{-30}}
 \end{aligned} \tag{38}$$

$$\begin{aligned}
 H &= 78.1266 \times 10^{-30} \text{ cm}^{-1} \\
 H &= \frac{78.1266}{1.08} \text{ km/sec} / \text{Mpc} \\
 H &= 72.3 \text{ km/sec} / \text{Mpc}
 \end{aligned} \tag{39}$$

We calculate Hubble constant from our hyperbolic scale factor,

$$\begin{aligned}
 R_{\text{now}} &= \sqrt{3/8\pi\rho_{\text{now}}} \sinh t_{\text{now}} \sqrt{8\pi\rho_{\text{now}}/3} \\
 R_{\text{now}} &= \sqrt{3/8\pi \times 7.425 \times 10^{-60}} \\
 &\times \sinh \left[1.32587 \times 10^{28} \times \sqrt{8\pi \times 7.425 \times 10^{-60} / 3} \right] \\
 R_{\text{now}} &= 1.6 \times 10^{29} \times \sinh 0.08287 \text{ cm} = 1.3 \times 10^{28} \text{ cm} \\
 \dot{R} &= \cosh t_{\text{now}} \sqrt{8\pi\rho_{\text{now}}/3}
 \end{aligned} \tag{40}$$

$$\begin{aligned}\therefore \sinh 0.08287 &= \frac{1.3}{16} \\ \therefore \cosh 0.08287 &= \sqrt{1 + \left(\frac{1.3}{16}\right)^2} = 1.0103 \\ H &= \frac{\dot{R}}{R} = \frac{\cosh t_{\text{now}} \sqrt{8\pi\rho_{\text{now}}/3}}{\sqrt{3/8\pi\rho_{\text{now}}} \sinh t \sqrt{8\pi\rho_{\text{now}}/3}} \\ H &= \frac{1.0103}{1.3 \times 10^{28} \text{ cm}} = 77.72 \times 10^{-30} \text{ cm}^{-1} \quad (41) \\ H &= 72 \text{ km/s / Mpc} .\end{aligned}$$

4. Conclusions

-Assuming a flat universe, the existence of dark matter and that the law of gravitation is universal, two teams of astrophysicists at the Lawrence Berkeley National Laboratory found that the supernovae weren't brighter—and therefore nearer—than expected. They were dimmer—that is, more distant. The two teams both concluded that the expansion of the universe isn't slowing down. It's speeding up.

-The motion on straight line (or flat space) is uniform (not accelerated). So, if the universe is not flat as assumed, the accelerated expansion is not due a dark energy; it's an attribute of the curved spacetime.

-Mean while we exhibit a mathematical proof of the hyperbolic universe. The flat universe has not any mathematical underpinning. Although perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space (i.e. the Universe locally is approximately flat consistent with local observations), the apparent angular size of distant objects falls off much more rapidly, in fact exponentially. The Universe is globally hyperbolic.

-The negative pressure $p = -\rho_j$ is a property of the hyperbolic structure of the Universe. The hyperbolic structure of the spacetime— not the cosmological constant—causes the apparently observed accelerating expansion of the Universe.

-According to General Relativity, gravity is not a force (due to Newton), rather it's the curvature of the spacetime. Einstein's equivalence principle states: It's impossible to detect the gravitational field (curvature) locally (on small scales). The curvature dominates the large structure spacetime, hence it's the cause of the appeared accelerated expansion of the universe.

-The critical density of the flat universe is $\rho_c = 10^{-29} \text{ g.cm}^{-3}$ corresponding to $\Omega_c = 1$. The hypothetical critical density necessary for a flat universe requires mysterious ingredients: Dark Matter and Dark Energy that have never been observed.

-The current observed density of the Universe $\rho = 10^{-31} \text{ g/cm}^3$ consistent with a hyperbolic open universe. The hyperbolic universe consists of zero cosmological constant [16]. The hyperbolic universe doesn't need Dark Energy to account for the accelerated expansion.

-The hyperbolic time evolution equation of the universe

$$R = \sqrt{3/8\pi\rho_j} \sinh t \sqrt{8\pi\rho_j/3} \quad (42)$$

As a solution to the first dynamical equation of cosmology

$$\dot{R}^2 + k = (8\pi/3)\rho R^2 \quad (43)$$

When applied to the second dynamical equation of cosmology

$$2R\ddot{R} + \dot{R}^2 + k = -8\pi p R^2 \quad (44)$$

Predicts the equation of state of cosmology $p = -\rho_j$

-The equation of state of cosmology $p = -\rho_j$ is a property of the hyperbolic structure of the hyperbolic universe.

-The pressure less Universe describes a decelerating expansion state of the universe which is described by the energy tensor of matter for dust where $p = 0$. Hence, the zero pressure does not lead to a dusty universe. In fact zero pressure Universe is an empty space, since $\rho = 0$. In the presence of the pressure

$$\begin{aligned} \ddot{R} &= -\frac{4\pi}{3} \rho + 3p R \\ \because p &= -\rho \\ \therefore \ddot{R} &= \frac{8\pi}{3} R > 0 \end{aligned} \quad (45)$$

The expansion of the Universe is accelerated.

-The accelerated expansion of the universe is described by the motion through a curved hyperbolic spacetime. The hyperbolic universe does not need Dark Energy.

-The Hyperbolic Universe grows exponentially preserves a legitimate inflation covers the current observed large structure 1.3×10^{28} cm.

-We calculate Hubble constant for the Hyperbolic Universe associated with $\rho_{\text{observed}} = 3 \times 10^{-31} \text{ g/cm}^3$ from first Friedmann's equation, to be

$$\begin{aligned} H^2 &= \frac{\dot{R}^2}{R^2} = \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} \rho_{\text{observed}} - \frac{k}{R^2} \\ \because k &= -1 \\ H &= \sqrt{\frac{8\pi}{3} \rho_{\text{observed}} + \frac{1}{R^2}} \\ H &= 72.3 \text{ km/sec / Mpc} \end{aligned} \quad (46)$$

-We calculate Hubble constant from the time-density equation of the Hyperboic universe, to be

$$\begin{aligned}
 R &= \sqrt{3/8\pi\rho_{now}} \sinh t \sqrt{8\pi\rho_{now}/3} = 1.3 \times 10^{28} \text{ cm} \\
 \dot{R}_{now} &= \cosh t_{now} \sqrt{8\pi\rho_{now}/3} = 1.0103 \\
 H &= \frac{\dot{R}}{R} = \frac{\cosh t_{now} \sqrt{8\pi\rho_{now}/3}}{\sqrt{3/8\pi\rho_{now}} \sinh t \sqrt{8\pi\rho_{now}/3}} \\
 H &= \frac{1.0103}{1.3 \times 10^{28} \text{ cm}} = 77.72 \times 10^{-30} \text{ cm}^{-1} \\
 H &= 72 \text{ km/s / Mpc} .
 \end{aligned} \tag{47}$$

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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