Control System Design of Uncertain System without Model Based on Newton's Laws of Motion

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Abstract:
A control system for uncertain plan without model is designed based on Newton's laws of motion and Kalman filter in the paper. The stability and unbiased estimate of the state observer are proven by Kalman filter theory via suitable setting system parameters. The controller structures and parameters are designed by feedback linearization based on Newton's laws of motion. All parameters in the system are only rated to the desired transient process time of system output without controlled plan model. If the desired transient process time of system output can’t be defined by control engineer, then the process time is set, where is the control period (or sample time period of DCS. Both the simulation example and engineering application example demonstrated the fine control quality and robust performance of the design method in the paper for uncertain systems without model.

Keywords:
Newton’s Laws of Motion, Uncertain System, Kalman Filter, State Observer, Feedback Linearization, Robust performance

1. Introduction

Design of control systems needs a mathematics model of controlled plant in general, however the model is not exactly obtained and the model is uncertain, these limits led to the poor control system performance.

The paper designs a control system based on Newton's Laws of Motion without exact plant model. The control system is of good robust performance, and has self-adaptation ability to system uncertainty.

When a motion (response) velocity of controlled plant process (body) is greatly slower than velocity of light, we can describe its motion by using Newton's Laws of Motion.

Newton's Laws of Motion are described as follows:
First law: In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.

Second law: In an inertial reference frame, the vector sum of the forces $F$ on an object is equal to the mass $m$ of that object multiplied by the acceleration of the object:

$$F = ma \quad \text{(It is assumed here that the mass $m$ is constant)}$$

Third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

2. Newton's Laws of Motion and Kalman Filter[1][2][4][6][7][8]

The second order controlled plant process is a typical controlled plant process in control theory and engineering, it is represented as:

$$\dot{y} = f(y, \dot{y}, w(t)) + bu$$

(1)

where, $w(t)$ is an unknown random disturbance.

Its state equation is represented as:

$$\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= f(y_1, y_2, w(t)) + bu \\
y &= y_1
\end{align*}$$

(2)

The uniformly acceleration motion in Newton's Laws of Motion is:

$$\begin{align*}
S &= S_0 + V_0t + 0.5at^2 \\
\dot{S} &= V = V_0 + at \\
\ddot{S} &= V = a
\end{align*}$$

(3)

Where $S, V, a$ are respectively the position, velocity and acceleration of the body motion.

For a controlled plant process, assuming $z(1), z(2), \ldots, z(k)$, the $k$ measurement output data are obtained from the controlled plant output $y(k)$, sample/(control) period is $t_s$, the measurement equation is

$$z(k) = y(k) + v(k)$$

(4)

Where:

$$E[V(k)] = r_2$$

$$E[v(k)] = 0$$

$v(k)$ is white noise, we can estimate the controlled plant output $y(k)$ using Eq. (3) from $z(1), z(2), \ldots, z(k)$, when $t_s$ is very short,

$$\begin{align*}
\hat{y}(k) &= \hat{y}(k-1) + t_s \hat{\dot{y}}(k-1) + 0.5t_s^2 \hat{\ddot{y}}(k-1) \\
\hat{\dot{y}}(k) &= \hat{y}(k-1) + t_s \hat{\dot{y}}(k-1) \\
\hat{\ddot{y}}(k) &= \hat{\ddot{y}}(k-1)
\end{align*}$$

(5)
where \( \hat{y}(k), \dot{\hat{y}}(k) \) and \( \ddot{\hat{y}}(k) \) are respectively the estimated values of \( y(k), \dot{y}(k) \) and \( \ddot{y}(k) \) at time \( k \).

Let

\[
\hat{Y}(k) = \begin{bmatrix}
\hat{y}(k) \\
\dot{\hat{y}}(k) \\
\ddot{\hat{y}}(k)
\end{bmatrix}
\]

And

\[
\phi = \begin{bmatrix}
1 & t_s & t_s^2/2 \\
0 & 1 & t_s \\
0 & 0 & 1
\end{bmatrix}
\]

Eq. (5) can be written as:

\[
\hat{Y}(k) = \phi \hat{Y}(k-1)
\]

To improve estimate accuracy, the compensating for random disturbance is into to acceleration estimate \( \ddot{\hat{y}}(k) \):

\[
\ddot{\hat{y}}(k) = \ddot{\hat{y}}(k-1) + w(k-1)
\]

where : \( E[w(k)] = 0, E[w^2(k)] = \sigma_w^2 \)

Eq.(7) becomes:

\[
\hat{Y}(k) = \phi \hat{Y}(k-1) + \Gamma w(k-1)
\]

Where:

\[
\Gamma = \begin{bmatrix}
0 & 0 & 1
\end{bmatrix}
\]

The measurement equation (4) is written as:

\[
\hat{z}(k) = c \hat{Y}(k) + v(k)
\]

Where: \( c = [1 \ 0 \ 0] \)

\[
E[w(k)] = 0, E[v(k)] = 0, E[y(0)] = m, E[w(k)v^T(j)] = 0, E[\dot{y}^2(k)] = \sigma_{\dot{y}}^2, E[w^2(k)] = \sigma_w^2, E[y(0)] = m, V_{arx}[y(0)] = P_0
\]

Kalman filter theory can be used for state equation (9) and measurement equation (10), \( \phi \) and \( \Gamma \) are constant matrices.

We calculate,
\[ (\phi, c) = [c^\tau | \phi^\tau c^\tau | (\phi^\tau)^2 c^\tau] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & t_s & 2t_s \\ 0 & t_s^2/2 & 2t_s^2 \end{bmatrix} \]

\[ (\phi, \Gamma) = [\Gamma | \phi \Gamma | \phi^2 \Gamma] = \begin{bmatrix} 0 & t_s^2/2 & 2t_s^2 \\ 1 & 1 & 1 \end{bmatrix} \]

\( \text{rank}(\phi, \Gamma) = 3 \), \( \text{rank}(\phi, c) = 3 \); so discrete systems (9) and (10) are completely controllable and observable, when \( t_s \) is very short and filtering time is very long, the covariance matrix,

\[ \lim_{k \to \infty} P(k) = P \]

Gain matrix

\[ \lim_{k \to \infty} K(k) = K^*, \]

\[ K(k) \to K^* = [\alpha, \beta, \gamma]^\top. \]

We have the following time discrete observer to an unknown controlled plant in Kalman filter theory.

\[ \begin{align*}
\dot{\hat{y}}(k) &= \hat{y}(k-1) + t_s \dot{\hat{y}}(k-1) + 0.5t_s^2 \ddot{\hat{y}}(k-1) + \alpha t_s (z(k) - \hat{y}(k-1)) \\
\dot{\hat{y}}(k) &= \hat{y}(k-1) + t_s \dot{\hat{y}}(k-1) + \beta t_s (z(k) - \hat{y}(k-1)) \\
\ddot{\hat{y}}(k) &= \ddot{\hat{y}}(k-1) + \gamma t_s (z(k) - \hat{y}(k-1)) \end{align*} \] (11.1)

Because Kalman filter is unbiased estimate, so the observer Eq. (11) based on Kalman filter also is unbiased estimate.

Let \( \hat{y}_1(k) = \hat{y}(k), \hat{y}_2(k) = \dot{\hat{y}}(k), \hat{y}_3(k) = \ddot{\hat{y}}(k) \) in Eq. (11), and Eq. (11) is operated as follows:

\[ \begin{align*}
(\hat{y}_1(k) - \hat{y}_1(k-1))/t_s &= \hat{y}_2(k-1) \\
+0.5t_s \hat{y}_3(k-1) + \alpha (z(k) - \hat{y}(k-1)) \\
(\hat{y}_2(k) - \hat{y}_2(k-1))/t_s &= \hat{y}_3(k-1) \\
+\beta (z(k) - \hat{y}(k-1)) \\
(\hat{y}_3(k) - \hat{y}_3(k-1))/t_s &= \gamma (z(k) - \hat{y}(k-1)) \end{align*} \] (11.2)

Let \( t_s \to 0 \) in Eq. (11.2), we have the time continuous observer

\[ \begin{align*}
\dot{\hat{y}}_1 &= \hat{y}_2 + \alpha (z - \hat{y}) \\
\dot{\hat{y}}_2 &= \hat{y}_3 + \beta (z - \hat{y}) \\
\dot{\hat{y}}_3 &= \gamma (z - \hat{y}) \end{align*} \] (12)

where,
Relating Eq. (2), control variable $bu$ is added to Eq. (12), Eq. (12) is became

$$\begin{bmatrix}
\dot{\hat{y}}_1 \\
\dot{\hat{y}}_2 \\
\dot{\hat{y}}_3 \\
\end{bmatrix} = \begin{bmatrix}
\hat{y}_2 + \alpha(z - \hat{y}) \\
\hat{y}_1 + \beta(z - \hat{y}) + bu \\
\gamma(z - \hat{y}) \\
\end{bmatrix}$$

(13.2)

Eq.(13) is written as:

$$\begin{bmatrix}
\dot{\hat{Y}}_1 \\
\dot{\hat{Y}}_2 \\
\dot{\hat{Y}}_3 \\
\end{bmatrix} = A \begin{bmatrix}
\hat{Y}_1 \\
\hat{Y}_2 \\
\hat{Y}_3 \\
\end{bmatrix} + Bu_0$$

(13.3)

Where

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \alpha & 0 \\
\beta & b \\
\gamma & 0 \end{bmatrix}$$

(13.4)

(13.5)

$A$ is represented in Eq.(13.1)

$$\begin{bmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\hat{y}_3 \\
\end{bmatrix} = \begin{bmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\hat{y}_3 \\
\end{bmatrix}$$

$t_i$ is sampling time, and $u$ is the control input.

Eq.(13.2) is the time continuous observer of an unknown controlled plant with control input $u$ for the control system represented in Eq. (2). Eq. (13.2) is simply called ONLM (Observer based on Newton’s laws of motion).

We have the following observer system for a controlled plant process with an acceleration $a = 0$,

$$\begin{bmatrix}
\dot{\hat{y}}_1 \\
\dot{\hat{y}}_2 \\
\hat{y} \\
\end{bmatrix} = \begin{bmatrix}
\hat{y}_2 + \alpha(z - \hat{y}) + bu \\
\hat{y}_1 + \beta(z - \hat{y}) \\
\hat{y} \\
\end{bmatrix}$$

We have the following observer system for a controlled plant process with varying acceleration $a$.
3. Control System Design of Uncertain System Without Model based on Newton's Laws of Motion (CSNLM) [1][2][4][6][7][8]

The designed Control System of Uncertain System without Model based on Newton's Laws of Motion is shown in Figure 1.

**Figure 1.** The designed Control System based on Newton's Laws of Motion.

### 3.1. Design Desired System Output Locus

In Figure 1, we have the transfer function between desired system output locus \( r(s) \) and control input \( u(s) \) in Eq. (14.1) by setting suitable parameters \( c, d \).

\[
\frac{r(s)}{u(s)} = \frac{1}{(cs^2 + ds + 1)} \quad (14.1)
\]

The control system output \( Y \) monotone smooth tracks control input \( u \). The time \( T \) when \( Y \) arrives at 98% \( u \) is defined as the desired time of the transition process of control system output \( Y \).

Eq. (14.1) state equation is represented as:

\[
\begin{align*}
\dot{r}_1 &= r_2 \\
\dot{r}_2 &= -dr_2/c - r_1/c + u/c \\
&= f(r_1, r_2) + r_r u \\
&= r = r_1 \\
\end{align*}
\]

Where,

\[
r_r = 1/c = w_n^2 \quad (14.3)
\]
is gain of desired system output locus , and \( w_n \) is natural frequency of desired system output locus, which represents system operation speed.

The desired time \( T \) of the transition process of control system output \( Y \) is related to \( w_n \).

\[
T = f\left(1 / w_n \right) \quad (14.4)
\]

General, \( T = nt \), \( n \in (800, 1200) \), \( T \) implies that control system arrives its desired value after run \( n \) times at control period \( t \).

3.2. Design State Observer (ONLM) [1][2][4][6][7][8]

The state observer is designed by Eq. (13.1).

Its characteristic equation by Eq. (13.1)

\[
m_a(s) = \left| sI - A \right| = s^3 + \alpha s^2 + \beta s + \gamma
\]

If \( \alpha > 0 \), \( \beta > 0 \), \( \gamma > 0 \) and \( \alpha \beta > \gamma \)
then time continuous observer system (13.1) is stable, according to Routh and Hurwitz's stability criterion.

Let \( m_a(s) = (s + s_i)^3 = 0 \), its 3 solutions at \(-s_i \) and \( s_i > 0 \) then

\[
\alpha = 3s_i, \beta = 3s_i^2, \gamma = s_i^3
\]

Suitable select \( s_i \) so that \( s_i = f_{11}(1/T, a_{11}) \), then

\[
\alpha = 3s_i = f_{11}(a_{11}, 1/T) = a_{11} / T;
\]
\[
\beta = 3s_i^2 = f_{12}(a_{12}, 1/T^2) = a_{12} / T^2
\]
\[
\gamma = s_i^3 = f_{13}(a_{13}, 1/T^3) = a_{13} / T^3
\]

3.3. Design Controller (CNLM) [2][3][4][5][6][8]

From Eq.(1) and Newton's Laws of Motion , we have the acceleration,

\[
a = \ddot{y} = f(y, \dot{y}, w(t)) + bu
\]

(16.1)

From Eq.(16.1), the control force \( u \) is obtained,

\[
u = (a - f(y, \dot{y}, w(t))) / b
\]

(16.2)

Take \( u \) in Eq.(16.2) to Eq.(2), we have:

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= a \\
y &= y_1
\end{align*}
\]

(17.1)

The transfer function between system output \( y \) and acceleration \( a \) for Eq.(17.1)

\[
y(s) / a(s) = g_{y,a}(s) = C^T [sI - A]^{-1}B = 1 / s^2
\]

(17.2)
Where,

\[
C' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

The transfer function \( g_{ys}(s) \) is a double integration function, so that poles position of the system can be allocated by acceleration \( a \). Eq.(16.2) is called as feedback linearization.

Under assuming \( y_3 = f(y_1, y_2, w(t)) \), The system Eq.(2) is converted to

\[
\begin{align*}
y_1' &= y_2 \\
y_2' &= y_3 + bu \\
y_3' &= f'(y_1, y_2, w(t)) \tag{18}
y &= y_1
\end{align*}
\]

Because the system Eq.(13.2) is an unbiased estimate system, comparing Eq.(18) with Eq.(13.2), we have:

\[
\begin{align*}
\hat{y}_1 &\rightarrow y_1 = y \\
\hat{y}_2 &\rightarrow y_2 = y'' \\
\hat{y}_3 &\rightarrow y_3 = f'(y_1, y_2, w(t))
\end{align*}
\]

so the \( \hat{y}_3 \) in Eq.(13) unbiased estimates the uncertain function \( f'(y_1, y_2, w(t)) \) in the system Eq.(2).

Design the acceleration \( \alpha \) in Eq. (17.2) as a controller

\[
\alpha = K_1(r - y) + K_2(\hat{r} - \hat{y}') \tag{19}
\]

Take \( \alpha \) in Eq.(19) to Eq.(17.2), because of unbiased estimate system Eq.(13) then we have:

\[
\begin{align*}
\hat{y} &= (K_1(r - \hat{y}_1) + K_2(\hat{r} - \hat{y}_2)) / s^2 \\
&= (K_1(r - \hat{y}) + K_2(rs - \hat{y}s)) / s^2.
\end{align*}
\]

The transfer function \( H(s) \) between control system output \( \hat{y} \) and control command \( r \)

\[
\hat{y}' \, r = H(s) = (K_1 + K_2s) / (s^2 + K_2s + K_1)
\]

The closed loop system characteristic equation

\[
m_{cl}(s) = (s^2 + K_2s + K_1)
\]

Design poles position at \(-s_2\), and \( s_2 > 0 \), for the closed loop system characteristic equation

\[
m_{cl}(s) = (s + s_2)^2 = s^2 + 2s_2s + s_2^2
\]

so, \( K_1 = s_2^2, K_2 = 2s_2 \),

Suitable select \( s_2 \) so that,

\[
K_1 = s_2^2 = f_{22}(a_{22}, 1/T^2) = a_{22} / T^2
\]
\[ K_2 = 2s_2 = f_{21}(a_{21}, 1/T) = a_{21}/T \]

to meet the performance target of the control system.

Comparing \( b \) in Eq.(2) with \( r \) in Eq.(14.2), we can estimate \( b \) in Eq.(2) by using Eq.(14.1), Eq.(14.2), Eq.(14.3) and Eq.(14.4).
\[
\hat{b} = f_{31}(r) = f_{32}(w_n^2) = f_{33}(1/T^2) = a_3/T^2
\]

From Eq.(16.2), the control force function:
\[
u = (K_r(r - \hat{y}_1) + K_2(\hat{r} - \hat{y}_2) - f(\hat{y}_1, \hat{y}_2, w(t))/\hat{b}
\]
as
\[
\hat{y}_3 \to f(y_1, y_2, w(t))
\]
so
\[
u \approx (k_1(r - \hat{y}_1) + k_2(\hat{r} - \hat{y}_2) - k_3\hat{y}_3)
\] (20)

Where
\[
k_1 = K_1/\hat{b}\,, \, k_2 = K_2/\hat{b}\,, \, k_3 = 1/\hat{b}
\]
\[
k = [k_1\, k_2\, k_3]^T
\]

All designed parameters are only related to the desired transient process time \( T \) of system output without controlled plan model in the paper.

Eq.(20) is the designed controller in Figure 1, As \( \hat{y}_3 \to f(y_1, y_2, w(t)) \), so \( u \) includes \( -f(y_1, y_2, w(t)) \) and eliminates uncertain and disturbance factors of the system. The control system (CSNLM) improves control qualities and increases robust performance of system.

When \( T = nt \), is used to above calculations, the parameters \( \alpha, \beta, \gamma, k_1, k_2, k_3 \) and \( \hat{b} \) are calculated by \( t_s \).

4. Simulation and Engineering Application Examples

4.1. Simulation Example

The transfer function \( g(s) \) of a controlled plant process with 2 run cases:
\[
g_0(s) = 0.8/(0.0060s^2 + 0.170s + 1),
g_1(s) = 1.5/(0.21s^2 + 0.36s + 1)
\]
(The transfer function is only for demo, not in designing parameters).

Design parameters: \( T=1(s), \, t_s = 0.001(s)\, \, , \, n = 1000; \)
\[
\alpha = a_{11}/T = 1428/T = 1428, \, \beta = a_{12}/T^2 = 679728/T^2 = 679728, \\
\gamma = a_{13}/T^3 = 107850176/T^3 = 107850176, \\
K_1 = a_{22}/T^2 = 384.16/T^2 = 384.16, \, K_2 = a_{21}/T = 39.2/T = 39.2,
\]
\[ \hat{b} = a_1 / T^2 = 61.46 / T^2 = 61.46, \]
\[ k_3 = 1 / \hat{b} = 0.0163, \quad k_1 = K_1 / \hat{b} = 6.25, \quad k_2 = K_2 / \hat{b} = 0.637 / T = 0.637. \]

Using CSNLM in Figure 1, simulation is carried out.

The control system (CSNLM) outputs under unit step signal input are shown in Figure 2:

4.2. Engineering Application Example

The transfer function \( g(s) \) of a controlled plant process with 2 run cases in some power plant:

\[ g_0(s) = 0.25 / (711s^2 + 53s + 1), \]
\[ g_1(s) = 1.5 / (661s^2 + 51s + 1). \]

(The transfer function is only for demo, not in designing parameters).

Design parameters: \( T = 200(s), \quad t_s = 0.2(s), \quad n = 1000 \)

\[ \alpha = a_{11} / T = 1428 / T = 7.14, \]
\[ \beta = a_{12} / T^2 = 679728 / T^2 = 17, \]
\[ \gamma = a_{13} / T^3 = 107850176 / T^3 = 13.5, \]
\[ K_1 = a_{22} / T^2 = 384.16 / T^2 = 0.0096 \]
\[ K_2 = a_{13} / T = 39.2 / T = 0.1960 \]
\[ \hat{b} = a_1 / T^2 = 61.44 / T^2 = 0.0015 \]
\[ k_3 = 1 / \hat{b} = 650.7705, \quad k_1 = K_1 / \hat{b} = 6.25, \quad k_2 = K_2 / \hat{b} = 127.6. \]

Using CSNLM in Figure 1, simulation is carried out.

The control system (CSNLM) outputs under unit step signal input are shown in Figure 3:
5. Conclusion

A control system for uncertain plan without model is designed based on Newton's laws of motion and Kalman filter in the paper. The stability and unbiased estimate of the state observer are proven by Kalman filter theory via suitable setting system parameters. The controller structures and parameters are designed by feedback linearization based on Newton's laws of motion.

All parameters in the system are only rated to the desired transient process time $T$ of system output without controlled plan model. If the desired transient process time $T$ of system output can't be defined by control engineer, then the process time $T = nt_s$ is set, where, $n \in (800, 1200)$, $t_s$ is the control period(or sample time period of DCS).

Both the simulation example and engineering application example demonstrated the fine control quality and robust performance of the design method (CSNLM) for uncertain systems without model, and CSNLM is of generality in the paper.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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