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Stability Analysis of PID Controllers-based First-Order with Dead-Time Unstable System

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Abstract:

The paper mainly emphasizes on the stability analysis of PID controllers-based first-order with dead-time unstable system. The research problem in this study is to investigate the difficulties of unstable condition for unknown plant. The objective of this work is to solve the stability solution for first-order with dead-time unstable system. In the field of Industrial process control, the performance, robustness and real constraints of control systems become more important to ensure strong competitiveness. The proportional-integral-derivative (PID) control is the most popular control strategy for regulating industrial processes. Although the performance and robustness gained by using PID controllers are acceptable in numerous applications, the PID controllers rapidly deteriorate when the time delay contained in the process is much greater than the principal time constant. Time delay is the main source of instability for the control loop. Typically, most unstable delay processes in practical systems are of low order (1st or 2nd-order). In this paper, a method to stabilize the first order plus dead time unstable process is developed by using a PID controller. In this paper, Nyquist Stability Criterion is used. The Nyquist Stability Criterion method is simply the tuning of controller gain in order to avoid the instability caused by time delay and used as a stability analysis tool. The test result is shown by MATLAB programming such as Nyquist plot and step response curves.

Keywords:

Dead-time, First-order, Proportional-Integral-Derivative (PID), Second-order, Control System Design, Stability Analysis

1. Introduction

Over the past fifty years, in parallel with the development of computer and communication technologies, control technology has made numerous significant successes in many areas. Its broad applications include guidance and control systems for aerospace vehicles, supervision control systems in the manufacturing industries, industrial process control systems, and real-time communication control systems.

These applications have had an enormous impact on the development of modern society [1,2].

Control theorists and engineers have developed reliable techniques for modeling, analysis, design, testing. From the industrial perspective, the performance, robustness and real constraints of control systems become more important to ensure strong competitiveness. Among most unity feedback control structures, the proportional-integral-derivative (PID) controllers have been widely used in many industrial control systems [3].

The importance of PID control comes from its simple structure, convenient applicability and clear effects of each proportional, integral and derivative control. The primary problem associated with the use of PID controllers is tuning. Due to the longstanding use of PID controllers in a variety of industries, there exist many different methods to find suitable controller parameters. Stabilization is one of the key issues in control engineering. However, time delay is commonly encountered in industrial process systems. In industrial and chemical practice, there are some open-loop unstable processes in industry such as chemical reactors, polymerization furnaces and continuous stirred tank reactors. Such unstable processes coupled with time delay make control system design a difficult task [4,5].

The typical unstable delay processes in industrial process systems are of low order. PID controllers are the dominant choice in process control. In this paper, based on the Nyquist stability theorem; the stabilization of first-order unstable time delay processes is investigated.

The rest of paper is organized as follows. Section II points out the PID theory. Section III mentions the stability methods. Section IV expresses the implementation of this work. Section V presents the results of these studies. Section VI discusses the results in conclusion section.

2. Background Theory of PID Control

A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly [6,7,8].

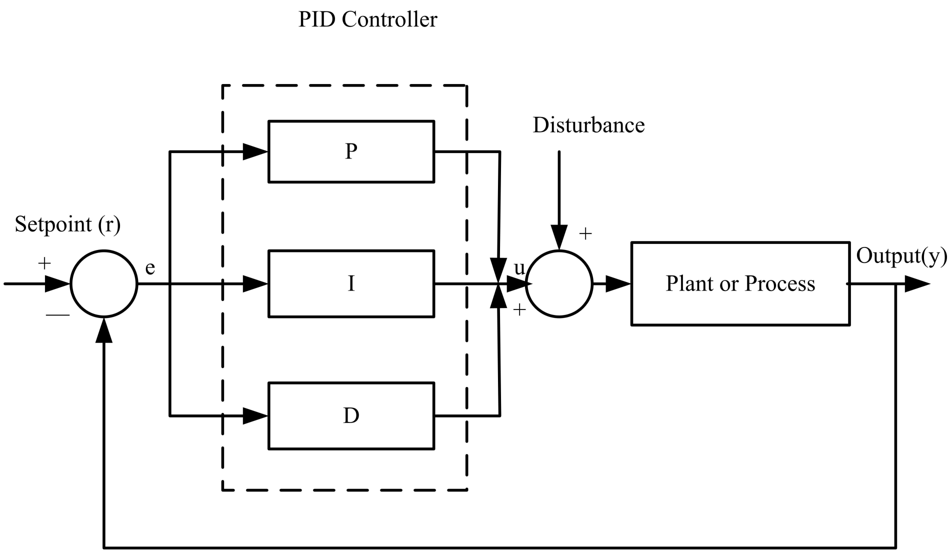

Figure 1. *Block diagram of PID controller [6].*

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PID control is the method of feedback control. The block diagram of PID controller is shown in Figure 1. The purpose of control is to make the process variable (y) follows the set-point value (r).

a. The Feedback Principle

Feedback works by measuring the current state of a physical system, determining how far the current state is from the desired state. Feedback can be used very effectively to stabilize the state of system [9].

b. The PID Response

The PID response contains proportional response, integral response and derivative response [10].

c. Action Modes of PID Controller

The combination of three functional elements (P, I, and D) used is called the "action mode" of the PID controller. Among them, the five listed in Table 1 are important in practice [11].

Table 1. *Action Modes of PID Controllers [6].*

Action mode	Element(s) used	Transfer function C(s)
Proportional (P)	P element only	$C(s) = K_p$
Integral (I) with $T_I = 1$	I element only	$C(s) = \frac{K_p}{s}$
Proportional-Integral(PI)	P and I elements	$C(s) = K_p \left[1 + \frac{1}{T_I s}\right]$
Proportional-Derivative (PD)	P and D elements	$C(s) = K_p \{1 + T_D D(s)\}$
Proportional-Integral-Derivative (PID)	All 3 elements	$C(s) = K_p \left\{1 + \frac{1}{T_I s} + T_D D(s)\right\}$

3. Stabilization Methods for Time Delay System

The following PID tuning formulas are considered:

a. Cohen–Coon (C–C) method

b. Ziegler–Nichols (Z–N) method

Pad'e Approximation method:

a. Smith Predictor method

b. Deparation method

c. Internal Model Control (ICM) method

d. Gain-phase margin (G-P) method

e. Integral-error based method

f. Nyquist Stability Criertion method

Among of these methods, Nyquist Stability Criertion method is used for paper. The Nyquist stability criterion, named after Harry Nyquist, is a graphical technique for determining the stability of a system. Because it only looks at the Nyquist plot of the open loop systems, it can be applied without explicitly computing the poles and zeros of either the closed-loop or open-loop system. Nyquist is one of the most general stability tests. For a system to be stable, all the poles of F(s) must lie in the left-half s-plane. Thus the roots of a stable system must lie to the left of the $j\omega$ -axis in the s-plane. The Nyquist stability theorem is now applied to the open-loop transfer function $Q(s) = C_i(s)G(s) = K\frac{N(s)}{s^v D(s)}e^{-Ls}, L\rangle 0$ which leads to the following theorem. Theorem means given the open-loop transfer function $Q(s)$ with unstable poles inside Nyquist contour, the closed –loop system is stable if and only if the Nyquist plot of Q(s) encircles the critical point, (-1,0), P^+ times anticlockwise.

4. Implementation of PID Controllers

For any open-loop transfer function Q(s) with P+ unstable poles inside the Nyquist contour, the closed-loop system is stable if and only if the Nyquist plot of Q(s) encircles the critical point, (-1,0), P+ times anticlockwise. P, PI,PD and PID are designed to control the first order plus dead time unstable process in this paper.

a. Design of P Controller

Open-loop transfer function Q(s) for P controller,

$$Q_P(s) = \frac{K_P}{s-1}e^{-Ls} \tag{1}$$

By property of Nyquist curve, subsequent analysis focuses on the positive frequently band and $\omega\rangle 0$ is always assumed unless otherwise indicated.

From mathematics calculation to find maximum and minimum limits for dead –time,

$$d_{\varphi_{Q_P}}(\omega) = -L\omega + \frac{1}{\omega^2+1} \tag{2}$$

From this equation, dead-time is chosen at less than 1.

Phase of transfer function,

$$-L\omega_{c_1} + \tan^{-1}(\omega_{c_1}) = 0 \tag{3}$$

Phase across over frequency is calculated. For calculating proportional gain, two conditions are used.

The open-loop transfer function has one unstable pole. Therefore,

$$P^+ = 1 \text{and} v = 0 \tag{4}$$

$$\therefore K\langle -1 (\because P^+ = 1 \text{and} v = 0) \tag{5}$$

The transfer function $Q(s)$ has no integrator. Therefore, the proportional gain is calculated from s=0.

The stability gain for P controller is bounded by the following.

$$1\langle K_P \langle \sqrt{1+\omega_{c_1}^2} \tag{6}$$

When dead time is chosen 0.5, the range of proportional gain is bounded by $1 \langle K_P \langle 2.3536$.When dead time is chosen 0.3, the range of proportional gain is bounded by $1 \langle K_P \langle 4.62$.

b. Design of PI Controller

Open-loop transfer function Q(s) for PI Controller,

$$Q(s) = K_P\left(1 + \frac{K_i}{s}\right)\frac{1}{s-1}e^{-Ls} \tag{7}$$

Due to the unknown value of K_i, the limit of the dead-time L is calculated by the following,

$$\varphi_{Q_{PI}}(\omega) = -L\omega - \tan^{-1}\left(\frac{K_i}{\omega}\right) + \tan^{-1}\omega - \pi \tag{8}$$

$$\varphi_{Q_P}(\omega) = -L\omega + \tan^{-1}\omega - \pi \tag{9}$$

$$\therefore \varphi_{Q_{PI}}(\omega) = \varphi_{Q_P}(\omega) - \tan^{-1}\left(\frac{K_i}{\omega}\right) \tag{10}$$

If dead- time for P controller is $L\langle 1$, dead-time for PI controller is assumed at $L\langle 1$. To choose K_i, the equation (11) is used .

$$\text{Max} \quad [\varphi_{PI}(\omega)] \rangle -\pi \tag{11}$$

To find K_P, the following limitation is used.

$$\lim_{\omega\to\infty} Q(j\omega) \langle 1 \tag{12}$$

Due to two values of phase across over frequency, the limit of K_P is achieved by the following.

$$\sqrt{\frac{1+\omega_{c_1}^2}{1+\left(\frac{K_i^2}{\omega_{c_1}^2}\right)}} \langle K_P \langle \sqrt{\frac{1+\omega_{c_2}^2}{1+\left(\frac{K_i^2}{\omega_{c_2}^2}\right)}} \tag{13}$$

Choosing dead time 0.5, the range of gains are bounded by $K_i \langle$ 0.309 and $1.1066 \langle K_P \langle 2.2516$.

Choosing dead time 0.3, the range of proportional gain is bounded by $K_i \langle$ 0.546 and $1.12 \langle K_P \langle 4.3$

c. Design of PD Controller

Open-loop transfer function Q(s) for PD Controller is

$$Q(s) = K_P(1 + K_d s)\frac{1}{s-1}e^{-Ls} \tag{14}$$

From mathematics calculation for maximum and minimum limit for K_d,

$$\varphi_{Q_{PD}}(\omega) = -L\omega + \tan^{-1}(K_d) + \tan^{-1}\omega - \pi \tag{15}$$

$$\frac{d\varphi_{Q_{PD}}(\omega)}{d\omega} = -L + \frac{1}{1+\omega^2} + \frac{1}{1+\omega^2 K_d^2} \times K_d \tag{16}$$

To find maximum limit for K_d,

$$\frac{d\varphi_{Q_{PD}}(\omega)}{d\omega} \rangle 0 \quad (\text{when } \omega = 0) \tag{17}$$

$$K_d \rangle \text{ L-1} \tag{18}$$

To find minimum limit for K_d , PD is no indicator and

The following limitation is used.

$$\lim_{\omega\to\infty} Q(j\omega) \langle 1 \tag{19}$$

$$\therefore L - 1 \langle K_d \langle 1 \tag{20}$$

From this equation, $L\langle 2$.

Lower limit of

$$K_P, K_P \rangle 1 \tag{21}$$

Upper limit of K_P,

$$\lim_{\omega\to\infty} Q(j\omega) \langle 1 \tag{22}$$

Range of K_P,

$$1 \langle K_P \langle \sqrt{\frac{1+\omega^2}{1+\left(K_d^2\omega^2\right)}} \tag{23}$$

Choosing dead time 1.5, the range of proportional gain and derivative gain are bounded by 0.5 $\langle K_d \langle$ 1 and 1 $\langle K_P \langle$ 1.108064467.

Choosing dead time 1, the range of proportional gain and derivative gain are bounded by 0 $\langle K_d \langle$ 1 and 1 $\langle K_P \langle$ 1.53.

d. Design of PID Controller

Open-loop transfer function Q(s) of PID Controller,

$$Q(s) = K_P\left(1 + \frac{K_i}{s} + K_d s\right)\frac{1}{s-1}e^{-Ls} \tag{24}$$

To find dead time range, the following general form of open- loop transfer function,

$$Q(s) = C_i(s)G(s) = K\frac{N(s)}{s^v D(s)}e^{-Ls} \tag{25}$$

$$Q_{PID}(s) = K_P\frac{\left(K_d s^2 + s + K_i\right)}{s(s-1)}e^{-Ls}$$

$$D(s) = s - 1, N(s) = K_d s^2 + s + K_i, v = 1$$

From [5]

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$$H(s)=\frac{\frac{d^{m+1}}{d_s^{m+1}}\left[s^{v}D(s)e^{Ls}\right]}{e^{Ls}} \qquad (26)$$

M is degree of N(s).

$L\langle 2$ is chosen.

When PID controller is used, let K_d in the same range of PD, then there exists a sufficiently small positive K_i such that $\max(\varphi_{Q_{PID}})\rangle -\pi$. To find K_P, the equation:

$$\lim_{\omega\to\alpha} Q(j\omega)\rangle -\pi.$$

$$\therefore L-1\langle K_d\langle 1 \qquad (27)$$

Choosing dead time 1.5, the range of proportional gain and derivative gain are bounded by $0.5\langle K_d \langle 1$, $K_i\langle 0.06$ and $1\langle K_P \langle 1.108064467$.

Choosing dead time 1, the range of proportional gain and derivative gain are bounded by $0\langle K_d \langle 1$, $K_i\langle 0.3$ and $1.1\langle K_P \langle 1.55$.

5. Results of First Order Plus Dead Time Unstable Process Using PID Controllers

In this paper, this process is control by using PID controller. Nyquist stability criterion is used as the tool for stability analysis. MATLAB program is used to aid obtaining result plots. The results of stability analysis and the performances of the system are shown in this Chapter.

a. P Controller

Dead-time for P controller is calculated for 0.5 and 0.3. The best result is shown by Nyquist plot, and step response for dead-time 0.3 at Figure 2 and Figure 3.The curve is enclosed at -1 in the left-half s-plane. So, the system is stabled at dead time 0.3. The response curve is stabled at 5 seconds and until the last time.

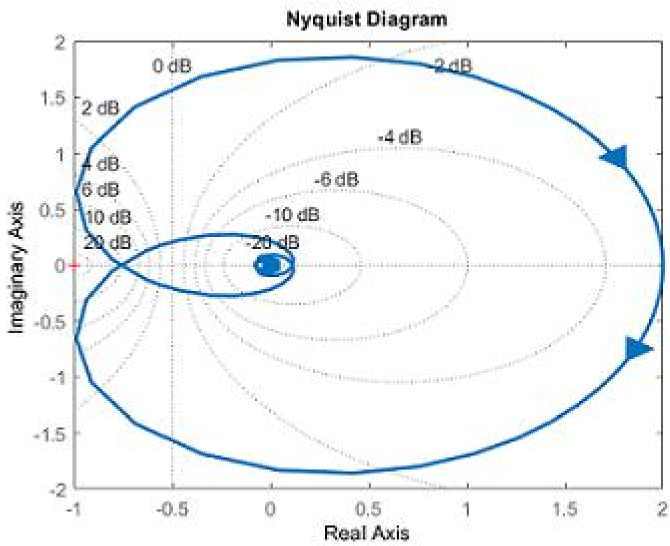

Figure 2. *Nyquist plot of P Controller at L=0.3.*

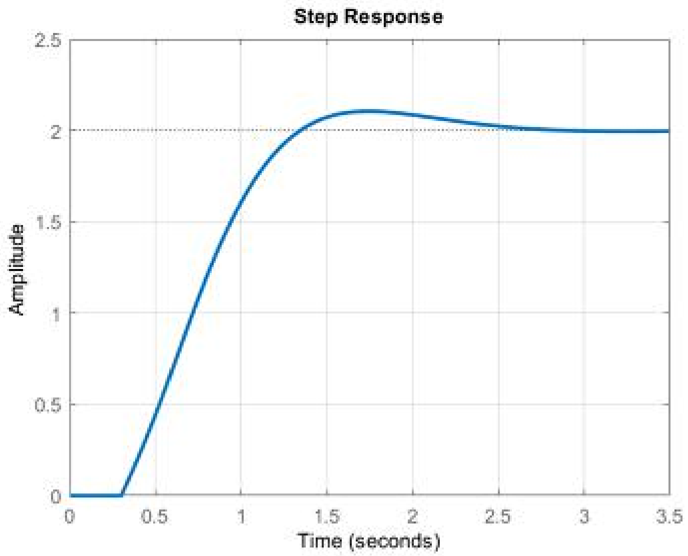

Figure 3. *Step response of P Controller at L=0.3.*

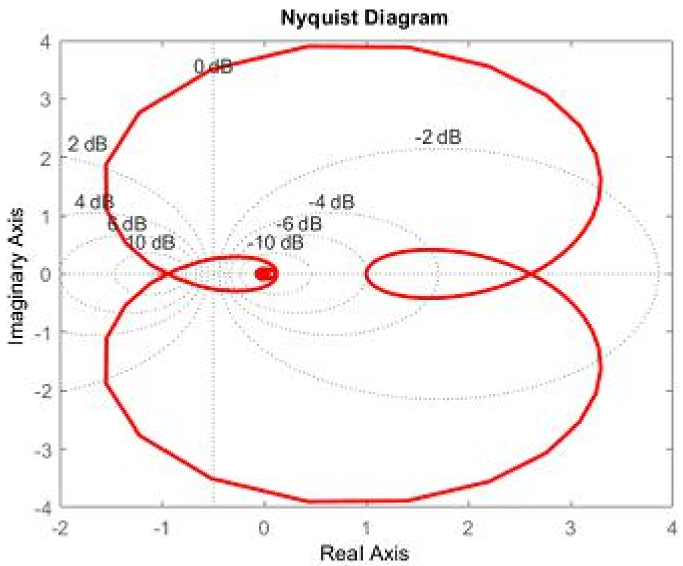

Figure 4. *Nyquist plot of PI Controller at L=0.3.*

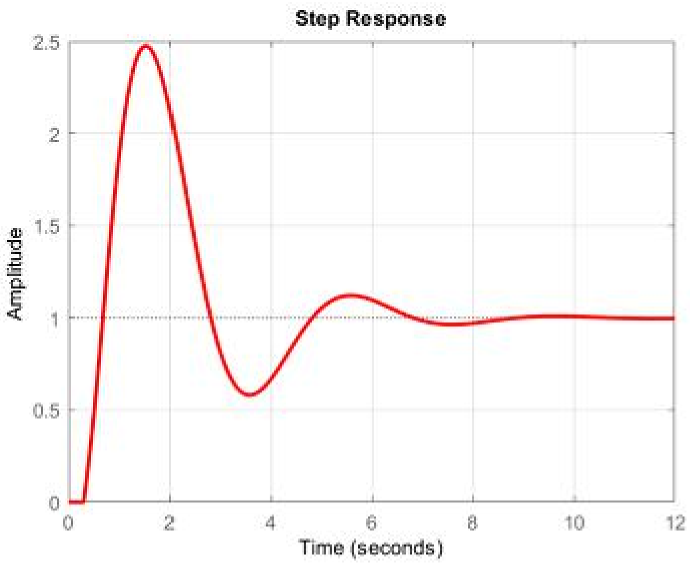

Figure 5. *Step response of PI Controller at L=0.3.*

b. PI Controller

Dead-time for PI controller is calculated for 0.5 and 0.3.The best result is shown by Nyquist plot, and step response for dead-time 0.3 at Figure 4 and Figure 5. The curve is enclosed at -1 in the left-half s-plane. So, the system is stabled at dead time 0.3 seconds. The response curve is stabled at 6.5 seconds and until the last time.

c. PD Controller

Dead-time for PD controller is calculated for 1 and 1.5.The best result is shown by Nyquist plots and response for dead-time 1 at Figure 6 and Figure 7. The curve is enclosed at -1 in the left-half s-plane. So, the system is stabled at dead time 1.The response curve is stabled at 10 seconds and until the last time.

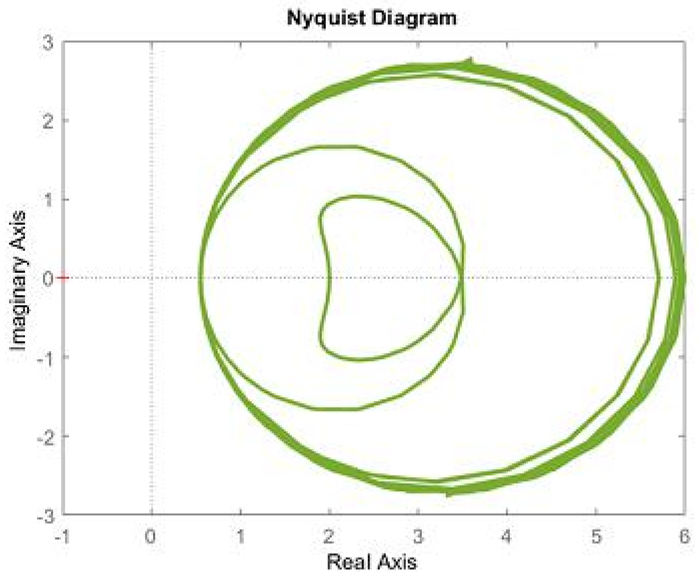

Figure 6. *Nyquist plot of PD Controller at L=1/*

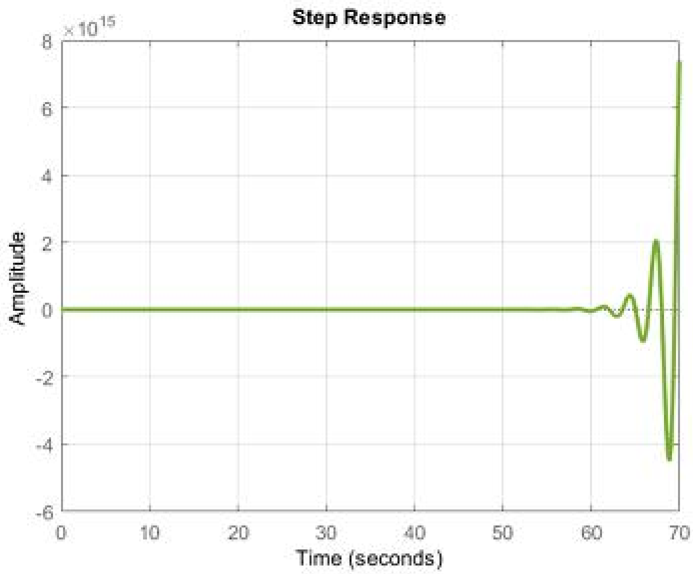

Figure 7. *Step response of PD Controller at L=1.*

d. PID Controller

Dead-time for PID controller is calculated for 1 and 1.5.The best result is shown by Nyquist plot, and step response for dead-time 1 at Figure 8 and Figure 9. The curve is enclosed at -1 in the left-half s-plane. So, the system is stabled at dead time 1. The response curve is stabled at 25 seconds and until the last time.

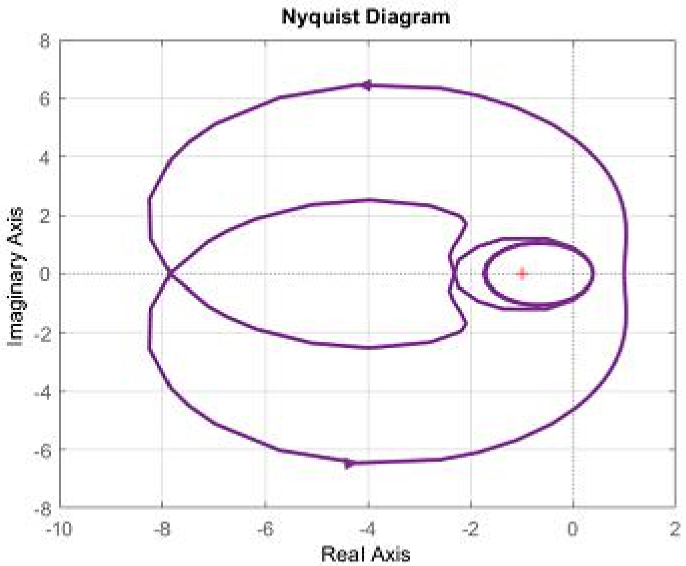

Figure 8. *Nyquist plot of PID Controller at L=1.*

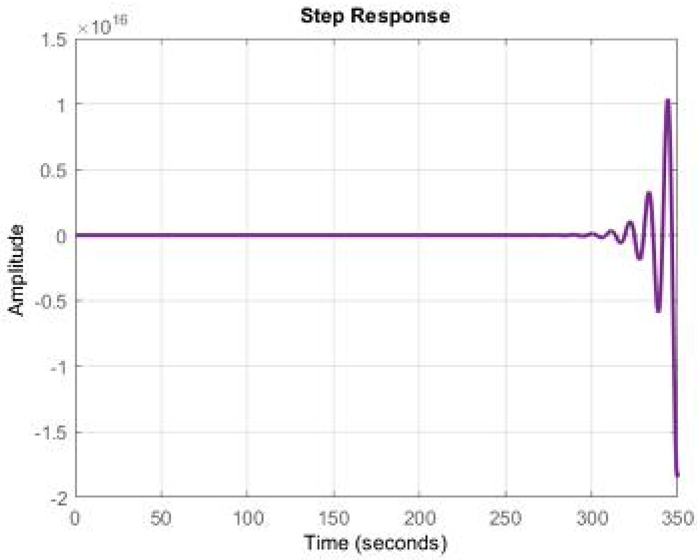

Figure 9. *Step response of PID Controller at L=1.*

6. Conclusion

Unstable process with dead time makes it difficult to design a control system. Most of the industrial processes are low order unstable process with time delay. Many methods are used for stable to system. Nyquist Stability theorem is used to tune this paper .Nyquist Stability theorem is shown that the system is stable by Nyquist plot. It is clear than other methods. The Nyquist plot is shown by MATLAB programming to easy. In many controllers is the best for first order plus dead time unstable process .It has suitable overshoot and start at 1 for stability.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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