

Error Decomposition and Application of Weighted Geometric Mean Combination Forecasting Model

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Abstract:

The ever-changing external environment drives people to be eager to grasp the trend of future things, and scientific forecasting methods come into being. Examined from the perspective of multiple sources of information, combined prediction is more comprehensive and accurate than single prediction. Currently, most studies on combination forecasting method focus on the selection of optimal models and weighting coefficients, but few literatures have explored the decomposition of errors in combination forecasting method. In this paper, we will focus on the decomposition of the Root Mean Square Percentage Error(RMSPE) of the weighted geometric mean combined prediction model, and propose the concepts of mean deviation, prediction resolution coefficient, and variance of deviation error of the sub-prediction model based on consistency metrics, and give expressions for the decomposition of the root mean square percentage error metrics of combined prediction methods and each individual prediction method under certain conditions, respectively. In order to verify the feasibility of the theory, this paper selects 14 years of China's natural gas production as an example study, constructs three single prediction models, namely, linear regression model, BP neural network model and grey GM (1,1) model, and compares the prediction results with those of the weighted geometric mean combined prediction model, which shows that the combined prediction method significantly reduces the error and improves the prediction accuracy. This shows that the weighted geometric mean combination prediction model is an effective class of nonlinear combination prediction model.

Keywords:

Weighted Geometric Mean Combination Forecasting, Error Decomposition, RMSPE

1. Introduction

With the development of human civilisation, the rapid expansion of the population, the booming of science and technology and the economy, and the sharp increase in social uncertainties, forecasting, as a practical and necessary social practice, has

arisen and developed in response to the needs of society. There are more than 300 kinds of prediction methods developed so far, which can be broadly divided into qualitative prediction and quantitative prediction according to the different objectives and characteristics of prediction. Qualitative forecasting methods are based on human experience, reasoning and other subjective judgement-based forecasting methods. However, since qualitative forecasting is greatly influenced by human subjective factors, we prefer to use quantitative forecasting methods based on historical and present data. In fact, a single prediction method can only provide effective information from a certain perspective, and there is the possibility of ignoring other important information, which leads to a decrease in the accuracy of the prediction, Bates, J.M. and Granger, C.W.J. in 1969 for the first time put forward the concept of combined prediction, which is the same prediction of the same object using a different single prediction model to find the optimum under a certain criterion in the form of an appropriate weighted average, which is the same as the prediction model. It is to use different single prediction models for the same prediction object, and to find the optimal weighting coefficient under a certain criterion, so as to make full use of the information provided by various single prediction methods, and to achieve the purpose of improving the prediction accuracy. It can enhance the stability of prediction and has a high ability to adapt to future changes in the prediction environment. So far, a series of research results have been achieved at home and abroad, mostly focusing on the model as well as the determination of the weight coefficients, while this paper will explore the sources of error in model prediction, and the selection of single prediction methods with high accuracy and different categories is the key to improve the accuracy of prediction.

2. Weighted Geometric Mean Combination Forecasting Model

2.1. Basics

Combinatorial forecasting is the result of single forecasting that gives the corresponding mathematical model in the form of some set function. Let there be m single prediction methods, let f_i be the prediction value of the i -th single prediction method, $i=1,2,\dots,m$, and let f be the combined prediction value, then there are $f_i = \varphi(f_1, f_2, \dots, f_m)$, where φ is the set functional form. From the point of view of the constructed functional relationship, there are linear and non-linear combinatorial predictions.[1, 2]

Let k_i be the weighting factor of the m individual prediction methods. If the combination prediction is satisfied:

$$f_A = k_1 f_1 + k_2 f_2 + \dots + k_m f_m, \sum_{i=1}^m k_i = 1, k_i \geq 0, i = 1, 2, \dots, m$$

Then f_A is said to be a linear combination of predictions.

If the combination prediction is satisfied:

$$f_B = \prod_{i=1}^m f_i^{k_i}, \sum_{i=1}^m k_i = 1, k_i \geq 0, i = 1, 2, \dots, m$$

Then f_B is said to be a weighted geometric mean portfolio prediction model.

If the combination prediction is satisfied:

$$f_C = \frac{1}{\sum_{i=1}^m \frac{k_i}{f_i}}, \sum_{i=1}^m k_i = 1, k_i \geq 0, i = 1, 2, \dots, m$$

Then f_C is said to be a weighted harmonic mean portfolio forecasting model. f_B and f_C are nonlinear combinatorial prediction models.

Let the time series corresponding to the actual observations of a prediction problem be $X = \{x_t, t = 1, 2, \dots, N\}$, with m sub-prediction models to predict it, and let $X_i = \{x_{it}, t = 1, 2, \dots, N, i = 1, 2, \dots, m\}$ be the sequence of predicted values of the i -th sub-prediction model at the t -th moment. Then there is the following basic concept.

Definition 2.1.1

$$\hat{x}_t = \prod_{i=1}^m x_{it}^{w_i}, t = 1, 2, \dots, N \# \quad (2.1.1)$$

$\hat{x}_t$ is said to be the weighted geometric mean of the combined prediction at moment t . $\hat{X} = \{\hat{x}_t, t = 1, 2, \dots, N\}$ is said to be the sequence of combined predictions. Where $w = (w_1, w_2, \dots, w_m)^T$ is the weighting coefficient of the corresponding sub-prediction model in the combined prediction and satisfies normalisation and non-negativity, i.e.

$$\sum_{i=1}^m w_i = 1, w_i \geq 0, i = 1, 2, \dots, m \# \quad (2.1.2)$$

For ease of calculation, equation (1) is changed to the following logarithmic form:

$$\ln \hat{x}_t = \sum_{i=1}^m w_i \ln x_{it}, t = 1, 2, \dots, N \# \quad (2.1.3)$$

Definition 2.2

$$e_{it} = \ln x_t - \ln x_{it}, e_t = \ln x_t - \ln \hat{x}_t = \sum_{i=1}^m w_i (\ln x_t - \ln x_{it}) = \sum_{i=1}^m w_i e_{it} \# \quad (2.1.4)$$

e_{it} is called the corresponding error between the predicted value and the actual value of the i -th forecasting method at the t -th moment, e_t is the error between the combined predicted value and the actual value at the t -th moment, $i=1, 2, \dots, m, t=1, 2, \dots, N$. [3] Then the sum of the squares of the total forecast error is

$$E = \sum_{t=1}^N e_t^2 = \sum_{t=1}^N \left(\sum_{i=1}^m w_i e_{it} \right)^2 = \sum_{i=1}^m \sum_{j=1}^m w_i w_j \left(\sum_{t=1}^N e_{it} e_{jt} \right) \# \quad (2.1.5)$$

Obviously, formula (2.1.5) shows that E is a function of $w = (w_1, w_2, \dots, w_m)^T$, so it is denoted as $E(w)$.

2.2. Solution

The smaller the sum of squares of the errors of the combined predictions, the more effective that combined prediction model is. In this sense, the weighting vector of the sub-prediction model is expressed as the following mathematical planning model:

$$\begin{aligned} \min E(w) &= \sum_{i=1}^m \sum_{j=1}^m w_i w_j \left(\sum_{t=1}^N e_{it} e_{jt} \right) \\ \text{s. t. } &\begin{cases} \sum_{i=1}^m w_i = 1 \\ w_i \geq 0, i = 1, 2, \dots, m \end{cases} \# \end{aligned} \quad (2.2.1)$$

The optimal solution of model (2.2.1) is the optimal weight coefficient of this weighted geometric mean combination prediction model.

3. RMSPE Decomposition for Weighted Geometric Mean Portfolio Forecasting Models

In order to grasp the good or bad prediction effect of each model and evaluate the accuracy of the prediction effect, we usually select certain error indicators as reference [3]. The common evaluation indicators are Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE), Root Mean Square Error (RMSE) and **Root Mean Square Percentage Error (RMSPE)**. [5] Among them, although MAE and MAPE can be a better measure of the regression model, the existence of the absolute value causes the function to be not smooth and cannot be derived at certain points, so it can be considered to change the absolute value to the square of the residuals, which is the Mean Square Error. However, although MSE and RMSE are widely used and important metrics, there are still some drawbacks. This is because both use the mean error, and the mean is more sensitive to outliers, if the regressor is very irrational about the regression value of a point, then it has a larger error, which will have a greater impact on the value of the RMSE, i.e., the mean is non-robust. Therefore, in this paper, the decomposition study will be carried out using RMSPE, which solves the robustness compared to MSE and RMSE.

3.1. Basic Concepts and Properties

In order to facilitate the exploration of the decomposition of the weighted geometric mean portfolio forecasting method RMSPE, the following definitions as well as properties are introduced for this purpose (easily provable ones are not to be proved):

Definition 3.1.1

$$\bar{Y}_i = \frac{1}{N} \sum_{t=1}^N y_{it}, \bar{\hat{Y}} = \frac{1}{N} \sum_{t=1}^N \hat{y}_t, \bar{Y} = \frac{1}{N} \sum_{t=1}^N y_t \quad i = 1, 2, \dots, m \quad \# \quad (3.1.1)$$

$\bar{Y}_i$ is said to be the mean of the time series corresponding to the i -th single forecasting method, and $\bar{\hat{Y}}$ is the weighted geometric mean combination of the mean of the time series corresponding to the forecasting method, and $\bar{Y}$ is the mean of the actual value time series.

Property 3.1.1

$$\bar{\hat{Y}} = \sum_{i=1}^m w_i \bar{Y}_i \quad \# \quad (3.1.2)$$

which indicates that the logarithm of the mean of the time series corresponding to the weighted geometric mean combination of forecasting methods is the weighted arithmetic average of the logarithm of the means of the individual forecasting methods.

Definition 3.1.2

$$RMSPE_i = \frac{1}{N} \sqrt{\sum_{t=1}^N \left(\frac{y_{it} - y_t}{x_{it}} \right)^2} \quad i = 1, 2, \dots, m \quad \# \quad (3.1.3)$$

$$RMSPE = \frac{1}{N} \sqrt{\sum_{t=1}^N \left(\frac{\hat{y}_t - y_t}{\hat{y}_t} \right)^2} \quad \# \quad (3.1.4)$$

$RMSPE_i$ is the RMSPE of the i -th prediction method, and $RMSPE$ is the RMSPE of the weighted geometric mean combination prediction. [6]

Definition 3.1.3

$$BSPE_i = \frac{1}{N^2} \sum_{t=1}^N \left(\frac{Y_{it} - \bar{Y}}{\bar{Y}_i} \right)^2 \# \quad (3.1.5)$$

$$BSPE = \frac{1}{N^2} \sum_{t=1}^N \left(\frac{\bar{Y} - \bar{Y}}{\bar{Y}} \right)^2 \# \quad (3.1.6)$$

$BSPE_i$ is the mean deviation of the i -th single prediction method, and $BSPE$ is the mean deviation of the weighted geometric mean combination prediction.

Definition 3.1.4

$$cov'(Y_i, Y) = \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_{it} - \bar{Y}_i)(y_t - \bar{Y}) \quad i = 1, 2, \dots, m \# \quad (3.1.7)$$

$$cov'(\hat{Y}, Y) = \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (\hat{y}_t - \bar{\hat{Y}})(y_t - \bar{Y}) \# \quad (3.1.8)$$

$$V'(Y) = \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_t - \bar{Y})^2 \# \quad (3.1.9)$$

$cov'(Y_i, Y)$, $cov'(\hat{Y}, Y)$, $V'(Y)$ are the covariance between the i th predicted value series and the actual value series, the covariance between the predicted value series and the actual value series of the weighted geometric mean combination, and the variance of the actual value series, based on RMSPE respectively.

Property 3.1.2

$$cov'(\hat{Y}, Y) = \sum_{i=1}^m w_i cov'(Y_i, Y) \# \quad (3.1.10)$$

Definition 3.1.5

$$Sl_i = \frac{cov'(Y_i, Y)}{V'(Y)}, \quad i = 1, 2, \dots, m \# \quad (3.1.11)$$

$$Sl = \frac{cov'(\hat{Y}, Y)}{V'(Y)} \# \quad (3.1.12)$$

Sl_i is the resolution coefficient between the i -th predicted value sequence and the actual value sequence, and Sl is the resolution coefficient between the weighted geometric mean group and the actual value sequence.

Property 3.1.3

$$Sl = \sum_{i=1}^m w_i Sl_i \# \quad (3.1.13)$$

Definition 3.1.6

$$RPV_i = (1 - Sl_i)^2 V'(Y), \quad i = 1, 2, \dots, m \# \quad (3.1.14)$$

$$RPV = (1 - Sl)^2 V'(Y) \# \quad (3.1.15)$$

RPV_i is the variance of the i -th individual predicted value sequence against the actual value sequence due to insufficient discriminatory ability, and RPV is the variance of the weighted arithmetic mean combined predicted value sequence against the actual value sequence due to insufficient discriminatory ability.

Definition 3.1.7

$$u_{it} = (y_{it} - \bar{Y}_i) - Sl_i(y_t - \bar{Y}), \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \# \quad (3.1.16)$$

$$\hat{u}_t = (\hat{y}_t - \bar{\hat{Y}}) - Sl(y_t - \bar{Y}) \# \quad (3.1.17)$$

$U^i = \{u_{it}, t = 1, 2, \dots, N\}$ is the sequence of deviation errors between the i -th single predicted value and the actual value based on the resolution coefficient, and $\widehat{U} = \{\widehat{u}_t, t = 1, 2, \dots, N\}$ is the sequence of deviation errors between the predicted value of weighted geometrical mean combination and the actual value based on the resolution coefficient.

Property 3.1.4

$$\overline{U^i} = 0, i = 1, 2, \dots, m, \overline{\widehat{U}} = 0 \# \quad (3.1.18)$$

Definition 3.1.7

$$DPEV_i = V'(U^i), i = 1, 2, \dots, m \# \quad (3.1.19)$$

$$DPEV = V'(\widehat{U}), t = 1, 2, \dots, N \# \quad (3.1.20)$$

DEV_i is the variance of the deviation error of the i -th single prediction method based on the resolution coefficient, and DEV is the variance of the deviation error of the weighted geometric mean combination prediction method based on the resolution coefficient.

3.2. Decomposition of RMSPE

Theorem

$$RMSPE_i = (BSPE_i + RPV_i + DPEV_i)^{1/2} \# \quad (3.2.1)$$

where $BSPE_i$, RPV_i , $DPEV_i$ denote the deviation from the mean, the variance due to the lack of discriminatory ability, and the variance of the deviation error of the sequence of the single prediction method and the sequence of the actual values, on RMSPE respectively.

Proof: From the defining equation (3.1.3) of the RMSPE, we have

$$RMSPE_i = \frac{1}{N} \sqrt{\sum_{t=1}^N \left(\frac{y_{it} - y_t}{x_{it}} \right)^2} = \left[\frac{1}{N^2} \sum_{t=1}^N \left(\frac{y_{it} - y_t}{x_{it}} \right)^2 \right]^{\frac{1}{2}} \# \quad (3.2.2)$$

Then, prove the internal structure of the root sign for convenience.

$$\begin{aligned} \frac{1}{N^2} \sum_{t=1}^N \left(\frac{y_{it} - y_t}{y_{it}} \right)^2 &= \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} [(y_{it} - \bar{Y}_i) + (\bar{Y}_i - \bar{Y}) - (y_t - \bar{Y})]^2 \\ &= \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_{it} - \bar{Y}_i)^2 + \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (\bar{Y}_i - \bar{Y})^2 + \frac{1}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_t - \bar{Y})^2 \\ &\quad + \frac{2}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_{it} - \bar{Y}_i)(\bar{Y}_i - \bar{Y}) - \frac{2}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (\bar{Y}_i - \bar{Y})(y_t - \bar{Y}) \\ &\quad - \frac{2}{N^2} \sum_{t=1}^N \frac{1}{y_{it}^2} (y_{it} - \bar{Y}_i)(y_t - \bar{Y}) \\ &= V'(Y^i) + BSPE_i + V'(Y) + 0 - 0 - 2cov'(Y^i, Y) \\ &= V'(Y^i) + BSPE_i + V'(Y) - 2cov'(Y^i, Y) \end{aligned} \quad (3.2.3)$$

From formula (3.1.9), we get

$$\text{cov}'(Y^i, Y) = Sl_i \cdot V'(Y) \# \quad (3.2.4)$$

Substituting the equation (3.2.4) into the equation (3.2.3), we have

$$V'(Y^i) + BSPE_i + (1 - 2Sl_i)V'(Y) \# \quad (3.2.5)$$

From formula (3.1.14), we get

$$RPV_i = (1 - Sl_i)^2 V'(Y) = (1 - 2Sl_i + Sl_i^2) V'(Y) \# \quad (3.2.6)$$

$$(1 - 2Sl_i)V'(Y) = RPV_i - Sl_i^2 V'(Y) \# \quad (3.2.7)$$

From formula (3.1.18), we have $\bar{u}^i = 0$. By (3.1.19) and (3.2.4), we get

$$\begin{aligned} DPEV_i &= V'(U^i) = \frac{1}{N} \sum_{t=1}^N (u_{it} - \bar{U}^i)^2 = \frac{1}{N} \sum_{t=1}^N (u_{it})^2 = \frac{1}{N} \sum_{t=1}^N [(y_{it} - \bar{Y}_i) - Sl_i(y_t - \bar{Y})]^2 \\ &= \frac{1}{N} \sum_{t=1}^N (x_{it} - \bar{X}^i)^2 - \frac{2Sl_i}{N} \sum_{t=1}^N [(y_{it} - \bar{Y}_i)(y_t - \bar{Y})] + \frac{Sl_i^2}{N} \sum_{t=1}^N (y_t - \bar{Y})^2 \\ &= V'(Y^i) - 2Sl_i \text{cov}'(Y^i, Y) + Sl_i^2 V'(Y) \\ &= V'(Y^i) - 2Sl_i \cdot Sl_i V'(Y) + Sl_i^2 V'(Y) \\ &= V'(Y^i) - Sl_i^2 V'(Y) \end{aligned} \quad (3.2.8)$$

Combining equation (3.2.7) and equation (3.2.5), and noting equation (3.2.8), we have

$$RMSPE_i = (BSPE_i + RPV_i + DPEV_i)^{1/2} \# \quad (3.2.9)$$

Testify to the end. The conclusion stands.

Similar conclusions can be drawn for combinatorial forecasting methods by the same reasoning.

$$RMSPE = (BSPE + RPV + DPEV)^{1/2} \# \quad (3.2.10)$$

This decomposition shows that the source of reducing the mean square error of the combined forecasts is, on the one hand, to choose individual forecasting methods with higher accuracy and, on the other hand, to increase the variability between individual forecasting methods.

4. Case Study on China's Natural Gas Production

In order to illustrate the weighted geometric mean combination prediction method given in this paper, the example of literature [7] is used here, and the linear regression model, the BP neural network model and the grey GM (1, 1) model are established for prediction by using the raw data of China's natural gas production from 2001-2014, and the fitting results are shown in Table 1.

Table 1. Actual values of natural gas production and forecasts from various models and their errors.

Year	Actual Value	Linear Regression/Error	BP Neural Network/Error	Grey Forecast/Error	Combined Forecast/Error
2001	303.00	256.60 / 46.40	300.60 / 2.40	303.00 / 0.00	303.00 / 0.00

2002	327.00	333.20 / -6.20	321.80 / 5.20	343.20 / -16.20	327.90 / 0.90
2003	350.00	409.80 / -59.80	357.40 / -7.40	389.40 / -39.40	357.40 / 7.40
2004	415.00	486.40 / -71.40	414.40 / 0.60	441.80 / -26.80	441.16 / 26.16
2005	493.00	563.00 / -70.00	494.70 / -1.70	501.20 / -8.20	494.70 / -1.70
2006	586.00	639.50 / -53.50	589.00 / -3.00	568.60 / 17.40	571.51 / 14.85
2007	692.00	716.10 / -24.10	682.30 / 9.70	645.10 / 46.90	706.06 / 14.06
2008	761.00	792.70 / -31.70	767.80 / -6.80	731.90 / 29.10	738.44 / -22.56
2009	830.00	869.30 / -39.30	849.70 / -19.70	830.40 / -0.40	830.40 / 0.40
2010	948.00	945.90 / 2.10	935.10 / 12.90	942.00 / 6.00	945.90 / -2.10
2011	1025.00	1022.50 / 2.50	1028.40 / -3.40	1068.80 / -43.80	1025.89 / 0.89
2012	1067.00	1099.08 / 3.20	1167.20 / 4.10	1212.50 / -21.20	1099.08 / 32.08
2013	1210.00	1175.67 / -5.10	1297.80 / -5.10	1375.60 / 2.10	1241.98 / 31.98
2014	1329.00	1356.87 / 2.10	1383.50 / 4.10	1375.50 / 3.50	1350.21 / 21.21

Then use these three single prediction models to establish a variable weight combination prediction model, substituting the data of the above single prediction model into the solution model (2.2.1), and changing the constraints of the weights to allow the existence of negative numbers[4], the optimal weights are obtained by using the LINGO software calculation as follows: $w_1 = -0.039$, $w_2 = 1.0135$, $w_3 = 0.0244$. The results of the derived combined prediction model and the error with the actual value are shown in the above Table 1, after substituting the optimal weights into the model of this paper, the image of the combined prediction value of China's natural gas production is plotted using Matlab, see the following figure 1. The fitting results are good.

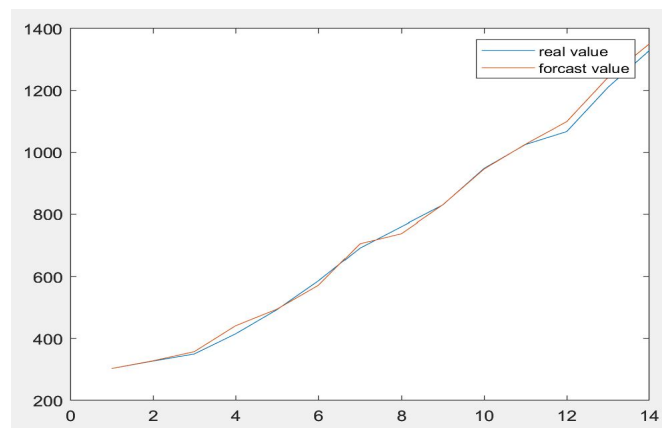


Figure 1. Actual and portfolio forecasts of natural gas production.

In order to reflect the effectiveness of the proposed combined prediction model, two error indicators, Mean Absolute Error(MAE) and the Mean Relative Error(MRE), are selected to evaluate the prediction effect.

$$MAE = \frac{1}{N} \sum_{t=1}^N |y_t - \hat{y}_t| \quad (4.1)$$

$$MRE = \frac{1}{N} \sum_{t=1}^N \left| \frac{y_t - \hat{y}_t}{y_t} \right| \quad (4.2)$$

The results are shown in the Table 2 below.

Table 2. Comparison of errors in the various forecasts.

	Linear Regression	BP Neural Network	Grey Forecast	Combined Forecast
Mean Absolute Error	38.806	22.521	42.271	12.566
Mean Relative Error	0.068	0.023	0.053	0.017

As can be seen from the above Table 2, the values of the error indicators of the single prediction models are larger than those of the base-optimal combination prediction of tax revenues, thus indicating that the combination prediction method proposed in this paper can effectively improve the prediction accuracy.

5. Conclusions

In this paper, based on the experience of previous research[8, 9], the error of the combination of prediction methods is decomposed, and the previous research is improved and explored in depth to obtain several aspects of the source of error, which can be used by decision makers as a basis for selecting better prediction models and methods. In order to further improve the theory of combinatorial prediction models and solve practical problems better, further exploration is needed in the future. [10] For example, it is necessary to add the theory of variable-weight combination prediction and discuss the combination of error and weight selection; as well as to discuss some other nonlinear combination prediction models, such as the weighted harmonic mean combination prediction model. From the examples in this paper, the prediction accuracy is still insufficient, and it can be surmised that the instability therein may be caused by external uncontrollable factors, such as natural disasters, national policies, and changes in the economic environment. Further research is needed on how to encapsulate these external factors into the model.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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