

The Construction Method of the Koch Snowflakes and Sierpinski Triangles and Their Implementation on MATLAB

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Abstract:

Fractals and self-similar structures are important concepts in fractal geometry, while the Koch snowflake and Sierpinski triangle are two common fractals. Both have strict self-similarity and play an important role in the study of quasi-crystal physical properties. In this paper, beginning with the generation mechanism of Koch snowflakes and Sierpinski triangles, the algorithms for generating Koch snowflakes and Sierpinski triangles are respectively designed (two generation methods, Chaos Game, and Delete Triangle Method, are applied to Sierpinski triangle), and finally implemented through MATLAB software programming, providing certain algorithms and programming ideas for generating these two kinds of fractals.

Keywords:

Ractal, Fractal Dimension, Self-Similarity, Koch Snowflake, Sierpinski Triangle

1. Basic Introduction To Fractals and Self-Similarity

In 1967, the French mathematician Benjamin Mandelbrot argued that the length of the British coastline could not be measured accurately. Ostensibly, as the length of the coastline is measured in smaller and smaller increments, the results become more precise. With this kind of thinking, the concept of the fractals was born.

For example, by removing all references on land, the shape of a 1,000-kilometre coastline would be similar or nearly similar to that of a 100-kilometre coastline. The concept is referred to as self-similarity.

In fractal geometry, the fractal dimension provides a ratio of statistical complexity indices and the fractal dimension would usually be a number between 1 and 2. A set is considered fractal if its theoretical fractal dimension exceeds its topological dimension.

2. The Koch Snowflake and Its Construction Method

The Koch curve is a classical iterative fractal curve, in fact, the first of the fractal curves to be described. According to the construction method of the Koch snowflake, a series of stages are involved in the building of the fractal iteratively. With the equilateral triangle being the first stage, outward curves on each side of the previous stage are added to generate smaller equilateral triangles to form the subsequent stages. Theoretically, the Koch snowflakes iterate indefinitely in this manner and a generative algorithm for reference could be as shown below.

- a. Draw an equilateral triangle.
- b. Trisect each side of the equilateral triangle and mark the trisections.
- c. Take the middle section of each side of the three equal parts as the side, outwardly drawn triangle, and remove the “middle section”.
- d. Repeat steps 2 and 3 above, keep drawing smaller triangles, and remove the “middle section”.
- e. Iterate until infinity, the final curve drawn would be the Koch snowflake.

Figure 1(a) indicates the Koch snowflake generated by the authors using MATLAB software and combining the above algorithms, starting from an equilateral triangle and going through 11 iterations. Figure 1(b) illustrates a part of the Koch snowflake fragment intercepted and enlarged by the authors, displaying an obvious self-similarity.

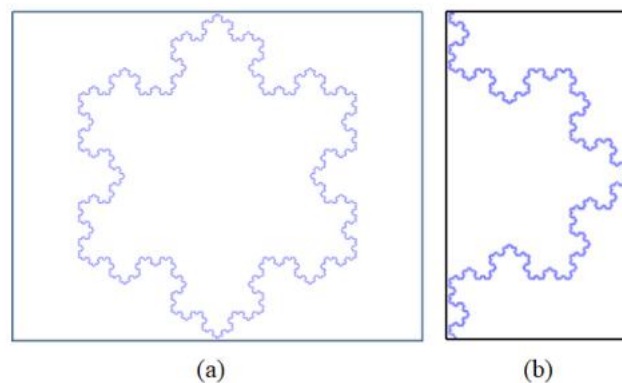


Figure 1. 11th generation Koch snowflake (a) and its enlarged fragment (b).

3. The Sierpinski triangle and Its Method of Construction

The overall shape of the periphery of the Sierpinski triangle is an equilateral triangle, and the interior is constantly and recursively subdivided into smaller equilateral triangles. There are several ways to construct the Sierpinski triangle. This section focuses on the deletion of the triangle method and the generation principle of the chaos game, subsequently, the relevant generation algorithms would be designed, and implemented through the MATLAB software.

3.1. Deleted Triangle Method

The deleted triangle method is a method of construction that begins with an equilateral triangle and removes a subset of the middle-most equilateral triangles by iterating repeatedly, as illustrated in Figure 2.

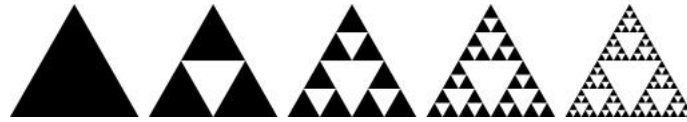


Figure 2. Delete Triangle Method Generation Principle.

This method has been well understood and the following generation algorithm is available for reference.

- a. First draw an equilateral triangle.
- b. Connecting the three midpoints, each side of this equilateral triangle is divided into equal parts, removing the middlemost one, and subdividing into four smaller equilateral triangles.
- c. For the remaining smaller equilateral triangles, the step 2 shall be repeated.
- d. Iterating until infinity, the resultant fractal would be the Sierpinski triangle.

Figure 3 indicates the Sierpinski triangle drawn by the authors based on the above algorithm after 10 iterations on the MATLAB software, where each of the deleted equilateral triangles is topologically an open triangle.

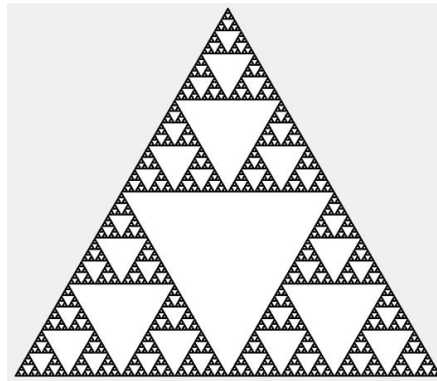


Figure 3. Sierpinski Triangle (Delete Triangle Method).

3.2. The Chaos Game

The exact principles of the Chaos Game, albeit not discussed in detail in this paper, illustrate that in this method, any point on the inside or outside of an equilateral triangle could become the starting point, and with some remaining points, result in the Sierpinski triangle being formed (Nonetheless, in case the starting point lies on the outline of the triangle, there would be no points remaining). After about a hundred points a short outline is formed while after close to a thousand points the details of the Sierpinski triangle begin to appear, and about 10^5 points would be needed to obtain a clear pattern.

Based on the principle of the chaos games, the following generative algorithm could be designed.

- a. Label the points p_1, p_2 and p_3 as the three corners of the Sierpinski triangle, with a random point v_1 .
- b. Using the formula $v_{n+1} = \frac{1}{2}(v_n + p_{r_n})$, where r_n is the random number 1, 2 or 3, plot the point v_1 to v_∞ .

Pertinent to note that in case the first point v_1 is considered to be a point on the Sierpinski triangle, consequently, all points v_n would be on the Sierpinski triangle, and vice versa. Figure 4 illustrates the Sierpinski triangle drawn by the author on the MATLAB software based on the above algorithm with the input parameter 10^7 .

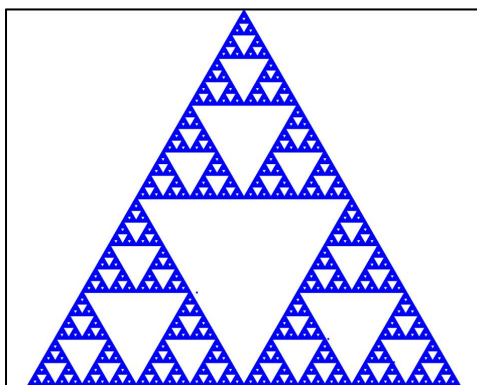


Figure 4. Sierpinski Triangle (The Chaos Game).

4. Conclusions

In this paper, a brief introduction of the concepts of fractals and self-similarity was given. Secondly, the construction methods of the two common self-similar structures (Koch snowflakes and Sierpinski triangles) were explained, and finally, the corresponding generating algorithms were designed and later these two common fractals were plotted successfully with the MATLAB software, respectively.

Needless to mention, a computer programme should be able to generate arbitrary generations of graphs and that only patterns after countless iterations would be the fractals in the true sense of the word. Nevertheless, since computers are unable to achieve countless iterations in reality, the images displayed by the computers would not be the true fractals, and the patterns of the true fractals exist only in ideal models, as the scaling described by the fractal dimensions does not last indefinitely, and the images exist only at the point of their smallest component.

The properties of the Koch snowflakes and the Sierpinski triangles have been well studied so far, moreover, the authors of this paper provide certain algorithms and ideas for generating the Koch snowflakes and the Sierpinski triangles.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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