

Construction Rules for Fibonacci Word and Its Implementation in MATL

Peiwen Du^{1*}

¹ School of Electrical and Computer Engineering, Guangzhou South College, Guangzhou, Guangdong, China

Email Address

dpw20002@sina.com (Peiwen Du)

*Correspondence: dpw20002@sina.com

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Abstract:

Models of quasi-periodic crystals are separated into one-, two-, and three-dimensional categories. The structural model of one-dimensional quasi-crystals is primarily based on the quasi-periodic Fibonacci sequence. In this paper, the authors primarily investigate the construction rules of one-dimensional quasi-periodic structures and design the generation algorithm for constructing Fibonacci strings using four methods: self-similar transformation method, formula method, iterative method, and high-dimensional projection method. On this basis, the one-dimensional point-line diagram and the two-dimensional quasi-periodic diagram (using the fifth generation as an illustration) are drawn.

Keywords:

Quasi-Crystals, One-Dimensional Quasi-Periodic Structures, Fibonacci Number, Fibonacci Word

1. Fibonacci Number and Fibonacci Word

The Fibonacci numbers are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,... or 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,... Each number in the series after the first two is the sum of the first two, and the first term is always either 0 or 1, depending on the starting point chosen. The series can therefore be described by the recurrence relation of formula (1-1):

$$F_n = F_{n-1} + F_{n-2} \quad (1-1)$$

The Fibonacci word is a specific sequence of binary digits that closely resembles the Fibonacci sequence's construction. If the assumption is "0" for "01", the formula (1-2) is as follows:

$$S_n = S_{n-1}S_{n-2} \quad (1-2)$$

Therefore, if the first two generations are known, it is possible to generate Fibonacci words starting from the third generation. The first six generations of Fibonacci words are shown in (1-3):

$$\begin{aligned}
 S_0 &= 0 \\
 S_1 &= 0\ 1 \\
 S_2 &= 0\ 1\ 0 \\
 S_3 &= 0\ 1\ 0\ 0\ 1 \\
 S_4 &= 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0 \\
 S_5 &= 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1
 \end{aligned}
 \tag{1-3}$$

Currently, Fibonacci-based constructions can simulate non-periodic physical systems (such as quasi-crystals).

2. Methods for Generating Fibonacci Word and Their Implementation in MATLAB

2.1. Iterative method

The iterative method involves replacing each '0' element with a pair of consecutive '01' elements and each '1' element with each '0' element. We can design and implement the generating algorithm using MATLAB programming based on this iteration rule. The following algorithm for generating is provided for reference.

Define the string and assign the value '0' to it, then assign this value to S_0 as the first generation of the Fibonacci word.

To achieve iteration, beginning with the second generation of Fibonacci words, replace the previous generation's '0' with the next generation's '01', the previous generation's '1' with the next generation's '0', and so on (a for loop can be used to achieve this iteration).

Output the iteration's result, which is the Fibonacci word for any generation.

Integrate the outputs to obtain the first n generations' Fibonacci word.

2.2. Formulaic approach

The nth element in a Fibonacci word can be found using formula (2-1):

$$1 + \lfloor n\varphi \rfloor - \lfloor (n+1)\varphi \rfloor \tag{2-1}$$

where φ is the golden ratio and $\varphi \approx 0.618$, commonly used in formula (2-2):

$$gold = \frac{1}{2 \cos\left(\frac{\pi}{5}\right)} \tag{2-2}$$

It is possible to determine the value of the nth Fibonacci word given an approximate value of 0.618 for φ using different positive integers (beginning with 1) for n. Moreover, in the Fibonacci word, the number of elements in each string generation is precisely equal to the Fibonacci series beginning with the second term, with the generative algorithm designed as follows for reference.

a. Use formula (2-2) to calculate the golden section number.

- b. Define two integers with values of 1 and 2 as the second and third terms of the Fibonacci numbers for subsequent generation of a series of Fibonacci words.
- c. Generate (using a for loop) a sequence of Fibonacci words using a recursive formula or generating rule for the Fibonacci number.
- d. Obtain the nth digit of the Fibonacci word using formula (2-1) (this can be done using a for loop).
- e. The Fibonacci word for the first n generations can be obtained by splicing and outputting the results from the preceding formulae in order.

2.3. Self-similar transformation method

Self-similar transformation is an essential technique for generating Fibonacci strings. If we assume '0' and S1 is '01', we have the following formula (2-3):

$$S_n = S_{n-1}S_{n-2} \quad (2-3)$$

With knowledge of the first two generations, it is possible to generate Fibonacci words beginning with the third generation. The available generative algorithm is designed as follows:

- a. Define two strings, '0' and '01', as the value of the sum, which corresponds to the first two generations of the Fibonacci word.
- b. The first two generations of the Fibonacci word are known, and the word can be generated using formulae (2-3) (which can be implemented using a for loop).
- c. Output the results to obtain any Fibonacci word generation.
- d. Integrate the output results to obtain the first n generations' Fibonacci word.

2.4. Higher dimensional projections

The Fibonacci word can also be represented by a sequence of cuts in a function with a slope $\frac{1}{\varphi}$ or $\varphi - 1$, where φ is the golden section and $\varphi \approx 1.618$. The analytical form of the function is shown in formula (2-4):

$$y = \varphi x \quad (2-4)$$

If you mark the intersection of the function with the X-axis when it takes a positive integer as '0' and the intersection with the Y-axis when it takes a positive integer as '1', and then arrange the marked '0' and '1' according to the positive direction of the order (along the positive direction of the X-axis and the positive direction of the Y-axis), you can obtain the first n generations of the Fibonacci word.

In the Fibonacci word, the number of elements of the string in each generation is precisely equal to the Fibonacci number beginning with the second term, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,..., and the low-dimensional quasi-crystal can be viewed as the projection of the high-dimensional crystals in the low-dimensional space, which can be used as a reference for the algorithm design of the following program.

- a. According to the function $y = \varphi x$, when $X = 1, 2, 3, \dots$, find the corresponding value of Y and record the coordinates according to the function, where X is a positive

integer, Y is the function of the corresponding positive integer, and φ takes the approximate value of 0.618.

b. According to the function $y = \varphi x$, determine the value of the corresponding X when Y takes a positive integer, and write down the coordinates (x, y) , respectively.

c. Using x-axis coordinates, sort the resulting coordinates from smallest to largest. Mark all coordinates with '0' for the X-axis integer and '1' for the Y-axis integer (the remainder is used for this).

d. Arrange '0' and '1' to obtain the Fibonacci word for any generation.

3. Plotting One-Dimensional Dot-Line Diagrams and Two-Dimensional Quasi-Dot Matrices

Based on generating the first n generations of Fibonacci words, it is possible to draw any generation's 1D dot line diagrams and 2D quasi-dot maps. Using the fifth-generation Fibonacci word as an illustration, the authors have successfully plotted the one-dimensional dot-line diagram and the two-dimensional quasi-dot matrix diagram, as depicted in Figures 3-1 and 3-2.

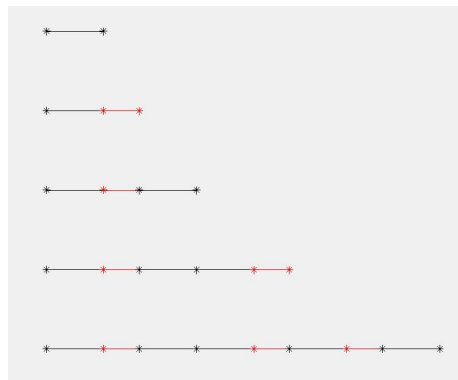


Figure 1. One-dimensional dot-line diagram of a series of Fibonacci words.

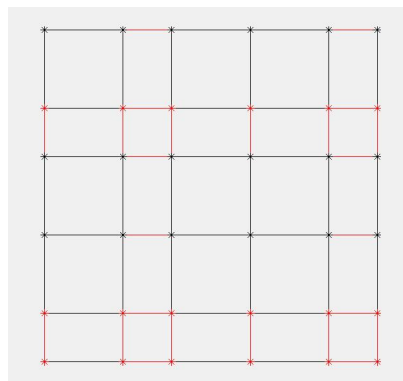


Figure 2. Two-dimensional quasi-dot plot of a series of Fibonacci words.

In a one-dimensional dot plot of Fibonacci words, the character '0' corresponds to a line segment of length 1 (denoted by a black line segment), while the character '1' corresponds to a line segment (denoted by a red line segment) with a length of the golden section ratio (with a value of approximately 0.618). On the other hand, in a two-dimensional quasi dot plot, the character '0' corresponds to a line segment L of length 1 (denoted by a black line segment), while the character '1' corresponds to a line segment S of length golden section ratio (denoted by a red line segment).

4. Conclusions

This thesis first briefly introduces the concepts of the Fibonacci number and Fibonacci word, which is a one-dimensional quasi-periodic structure in which the number of elements in each generation of its string is precisely equal to that of the Fibonacci number sequence beginning with the second term; secondly, it elaborates the generative principles of the four commonly used methods for generating the Fibonacci word, namely the iterative method, the formulaic method, the self-similarity transformation method, and the high-dimensional projection method. According to these methods, four generation algorithms for constructing the word are respectively designed. Based on the preceding information, the authors have drawn the one-dimensional dot-line diagram and the two-dimensional quasi-dot-matrix diagram of the Fibonacci word (using the fifth generation as an example).

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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References

- [1] Glen, A.; Simpson, J.; Smyth, W. More properties of the Fibonacci word on an infinite alphabet. *Theoretical Computer Science*, 2019, 795(C).
- [2] Barabash, M.G.; Kholyavka, M.Y.; Tytar, V.I. Periodic words connected with the Fibonacci words. *Karpats'ki Matematichni Publikacii*, 2016, 8(1).
- [3] Du, P.W. Construction Rules of P3 Penrose Tiling and Its MATLAB Implementation. *Academic Journal of Engineering and Technology Science*, 2023, 6(7).
- [4] Ramírez, L.J.; Rubiano, N.G.; Castro, D.R. A generalization of the Fibonacci word fractal and the Fibonacci snowflake. *Theoretical Computer Science*, 2014, 528.
- [5] M. S.R. R. Two-dimensional Fibonacci words: Tandem repeats and factor complexity. *Advances in Applied Mathematics*, 2023, 149.
- [6] Luca, D.A. Some Extremal Properties of the Fibonacci Word. *International Journal of Algebra and Computation*, 2013, 23(4).
- [7] Margenstern, M. Fibonacci words, hyperbolic tilings and grossone. *Communications in Nonlinear Science and Numerical Simulation*, 2015, 21(1-3).
- [8] László, N. Fibonacci words in hyperbolic Pascal triangles. *Acta Universitatis Sapientiae, Mathematica*, 2017, 9(2).
- [9] Yu, F.; Liu, H.Z.; Xu, Z.X. Some Results about Infinite Fibonacci Words. *Gamma*, 2013, 44.
- [10] Fraenkel, S.A.; Simpson, J. The exact number of squares in Fibonacci words. *Theoretical Computer Science*, 1999, 218(1).

- [11]Goldberg, L.; Fraenkel, S.A. Patterns in the generalized Fibonacci word, applied to games. *Discrete Mathematics*, 2018, 341(6).
- [12]Julien, C.; Idrissa, K. On the Complexity of the Generalized Fibonacci Words. *RAIRO - Theoretical Informatics and Applications*, 2022, 56.
- [13]Ramírez, L.J.; Rubiano, N.G. Biperiodic Fibonacci Word And Its Fractal Curve. *Acta Polytechnica*, 2015, 55(1).
- [14]Hoffman, T.; Steinhurst, B. Hausdorff Dimension of Generalizedfibonacci Word Fractals. *Fractals*, 2018, 26(1).



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