

Research on Calibration Coefficient of a Supersonic Carrier Flight Position Deviation Calculation Formula

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Abstract:

In this paper, linear and nonlinear fitting methods are proposed by analyzing the characteristics of a newly developed supersonic carrier in flight, which provides a scientific basis for accurately calculating the position deviation of the flight.

Keywords:

Supersonic Aircraft, Information Processing, Parameter Calibration

1. Introduction (Heading 1)

When the supersonic carrier flies, shock signal will be generated, sensor array will be constructed to detect the location of the sound source, and then the shooting location of the terrorist can be obtained. It is an effective method. The traditional method of calculating conical source point is obtained by d_B calculation [1], d_B is as follows:

$$d_B = C_1 \left[T_F - \frac{C_2}{CM_F} \right]^4 \frac{M_p^2 - 1}{M_p^3} \quad (1)$$

In this formula:

d_B : Distance from sound source transmitting point to detection point;

C : the speed of sound;

M_p : The Mach number of the flight carrier;

C_1 、 C_2 : Calibration coefficient of flight carrier;

T_F : Time for the carrier head and tail wave to sweep the detection sensor.

In calculation, C_1 and C_2 should be obtained through range calibration test, which is time-consuming, laborious and expensive. A new algorithm is developed in this paper, which can obtain the sound source position according to d_B calculation without passing the test [1,2], as show in Figure 1:

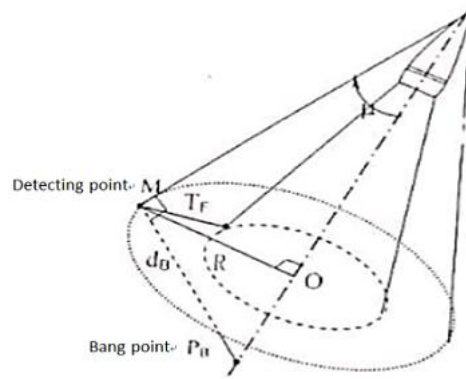


Figure 1. Schematic diagrams of conical waves.

While flight carrier at supersonic speeds through the atmospheric flight, the air disturbance and formation density variation of pressure wave, commonly known as the warhead wave. Each point on the headwave of flight carrier wave before and after the wave of the path is a vertex in the bullet in the head and tail cone. Wave front and wave after all present a cone (as shown in Figure 1), the wave front half Angle of cone is associated with the flight Mach number of bullets.

The miss distance R is measured in the calibration of the range, and the relationship between them can be obtained according to Figure 1:

$$R = d_B \cdot \cos \mu = \frac{(M_p^2 - 1)^{1/2}}{M_p} \cdot d_B \quad (2)$$

In formula (2), parameters R , d_B , T_F , M_p and C can be obtained through range tests. In order to obtain C_1 and C_2 , C_1 and C_2 can be solved using the data of two rounds. However, due to the deviation of each firing parameter and the disturbance in flight, the T_F value will be biased. In order to overcome this defect, the principle of least square method and data fitting of multiple rounds were used in the actual calibration test to accurately obtain C_1 and C_2 . When C_1 and C_2 are obtained, formula (2) the units of each quantity are: T_F is ms; The sound speed is ms^{-1} ; M_F is dimensionless; C_1 is $\text{m} \cdot \text{ms}^{-4}$; C_2 is mm.

Three fitting methods are adopted^[3,4,5]: a. Nonlinear fitting method; b. Linear fitting method 1; 3. Linear fitting method 2.

2. Nonlinear Fitting Method

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According to formula (1), d_B is a multivariate function of C_1 , C_2 , T_F , C and M_p .

$$d_B = F(C_1, C_2, T_F, C, M_p) = F(C_1, C_2, \bar{X}_k)$$

In the formula

$$\bar{X}_k = \begin{bmatrix} T_F \\ C \\ M_p \end{bmatrix}$$

Let a set of experiments to obtain n d_B measured values Y_K and the corresponding independent variable X_K , where $K = 1, 2, \dots, n$. First, C_1 and C_2 are given an initial value, denoted as C_i^0 , and C_i is their true value. Then there is

$$C_i = C_i^0 + \Delta_i \quad (i = 1, 2) \quad (3)$$

The function F is expanded in a first order near C_i^0

$$F(C_1, C_2, \bar{X}_k) \approx F_{k0} + \frac{\partial F_{k0}}{\partial C_1} \cdot \Delta_1 + \frac{\partial F_{k0}}{\partial C_2} \cdot \Delta_2$$

In the formula

$$F_{k0} = F(C_i^0, C_j^0, \bar{X}_k) \\ \frac{\partial F_{k0}}{\partial C_i} = \frac{\partial F(C_i^0, C_j^0, \bar{X}_k)}{\partial C_i} \quad (i = 1, 2)$$

Therefore, the sum of squares Q of the deviation between the calculated and measured values of each round is

$$Q = \sum_{k=1}^n [Y_k - F(C_1, C_2, \bar{X}_k)]^2 \approx \\ \sum_{k=1}^n \left[Y_k - \left(F_{k0} + \frac{\partial F_{k0}}{\partial C_1} \cdot \Delta_1 + \frac{\partial F_{k0}}{\partial C_2} \cdot \Delta_2 \right) \right]^2$$

Based on function minimization conditions

$$\frac{\partial Q}{\partial C_i} = 0 \quad (i = 1, 2)$$

So

$$\frac{\partial Q}{\partial C_i} = 2 \left[\Delta_1 \cdot \sum_{k=1}^n \frac{\partial F_{k0}}{\partial C_1} \cdot \frac{\partial F_{k0}}{\partial C_i} + \Delta_2 \cdot \sum_{k=1}^n \frac{\partial F_{k0}}{\partial C_2} \cdot \frac{\partial F_{k0}}{\partial C_i} - \sum_{k=1}^n (Y_k - F_{k0}) \cdot \frac{\partial F_{k0}}{\partial C_i} \right] = 0 \quad (i = 1, 2)$$

This is a system of linear equations

$$\begin{cases} \sum_{k=1}^n \left(\frac{\partial F_{k0}}{\partial C_1} \right)^2 \Delta_1 + \sum_{k=1}^n \left(\frac{\partial F_{k0}}{\partial C_1} \frac{\partial F_{k0}}{\partial C_2} \right) \Delta_2 = \sum_{k=1}^n \frac{\partial F_{k0}}{\partial C_1} (Y_k - F_{k0}) \\ \sum_{k=1}^n \left(\frac{\partial F_{k0}}{\partial C_1} \frac{\partial F_{k0}}{\partial C_2} \right) \Delta_1 + \sum_{k=1}^n \left(\frac{\partial F_{k0}}{\partial C_2} \right)^2 \Delta_2 = \sum_{k=1}^n \frac{\partial F_{k0}}{\partial C_2} (Y_k - F_{k0}) \end{cases}$$

The solution gives Δ_1 , Δ_2 , and then a new C_i value is obtained from formula (3). When the value Δ_i is large, the current C_i value can replace the original C_i^0 initial value. The above iteration process is repeated until the value Δ_i is small enough to be ignored, at which time the obtained C_i value is the desired.

3. Linear Fitting Method 1

It can be obtained from equations (1) and (2)

$$R = C_1 \cdot \left(T_F - \frac{C_2}{CM_P} \right)^4 \cdot \frac{(M_P^2 - 1)^{3/2}}{M_P^4} \quad (4)$$

Let's transform equation (4) into a function that fits linearly. Equation (4) raise both sides to the power of 4 and solve for T_F .

$$T_F = \frac{M_P}{C_1^{1/4} \cdot (M_P^2 - 1)^{3/8}} \cdot R^{1/4} + \frac{C_2}{CM_P}$$

$$T_F = K \cdot R^{1/4} + B \quad (5)$$

Because the calibration is carried out at the same transmitting distance, it can be approximated that M_P and C are constants, and the average value of a group of tests can be taken in actual calculation. In this way, T_F and $R^{1/4}$ are unary linear functions $\frac{M_P}{C_1^{1/4} \cdot (M_P^2 - 1)^{3/8}}$, is the constant of the slope K of the line, and C_2/CM_P is the constant of the intercept B of the line. Take $R^{1/4}$ as the horizontal coordinate and T_F as the vertical coordinate. n sets of tests get n coordinate points $(R_i^{1/4}, T_{Fi})$, $i=1, 2, \dots, n$. Using the least square method, the sum of squares of the deviations is first constituted by Formula (5)

$$Q = \sum_{i=1}^n (T_{Fi} - K \cdot R_i^{1/4} - B)^2$$

Given by the minimization condition

$$\frac{\partial Q}{\partial K} = 0 \quad \frac{\partial Q}{\partial B} = 0$$

a linear system of equations is obtained

$$\begin{cases} (\sum_{i=1}^n R_i^{1/2})K + (\sum_{i=1}^n R_i^{1/4})B = \sum_{i=1}^n (T_{Fi} \cdot R_i^{1/4}) \\ (\sum_{i=1}^n R_i^{1/4})K + n \cdot B = \sum_{i=1}^n T_{Fi} \end{cases}$$

Solve for the undetermined coefficients K , B , and you'll fit a line, and you'll find the corresponding C_1 and C_2 .

4. Linear Fitting Method 2

In actual testing, even at the same transmitting range, the storage rate of each test is different. Some storage speed deviation can reach $30 \text{ m}^* \text{s}^{-1}$. Therefore, considering M_P as a constant will bring some errors. Change formula (5) to

$$M_P \cdot T_F = \frac{1}{C_1^{1/4}} \cdot \frac{M_P^2}{(M_P^2 - 1)^{3/8}} \cdot R^{1/4} + \frac{C_2}{C}$$

$$K \left(\frac{M_P^2}{(M_P^2 - 1)^{3/8}} \cdot R^{1/4} \right) + B \quad (6)$$

We multiply M_P in front of T_F and $R_i^{1/4}$. $M_P \cdot T_F$ is the unary linear function of $\frac{M_P^2}{(M_P^2 - 1)^{3/8}} \cdot R^{1/4}$, which is the constant of the material rate K of the line, and C_2/C is the constant of the intercept B of the line. With $\frac{M_P^2}{(M_P^2 - 1)^{3/8}} \cdot R^{1/4}$ as the horizontal coordinate and $M_P \cdot T_F$ as the vertical coordinate, n coordinate points can be obtained through testing calibration test:

$$\left(\frac{M_{Pi}^2}{(M_{Pi}^2 - 1)^{3/8}} \cdot R_i^{1/4}, M_{Pi} \cdot T_{Fi} \right) \quad (i = 1, 2, \dots, n)$$

Using the least square method, the sum of squares of deviation is formed from formula (6)

$$Q = \sum_{i=1}^n \left(M_{Pi} \cdot T_{Fi} - K \left(\frac{M_{Pi}^2}{(M_{Pi}^2 - 1)^{3/8}} \cdot R_i^{1/4} \right) - B \right)^2$$

Obtained by minimizing conditions

$$\frac{\partial Q}{\partial K} = 0 \quad \frac{\partial Q}{\partial B} = 0$$

a linear system of equations is obtained

$$\begin{cases} \left[\sum_{i=1}^n \frac{M_{Pi}^2 \cdot R_i^{1/2}}{(M_{Pi}^2 - 1)^{3/4}} \right] K + \left[\sum_{i=1}^n \frac{M_{Pi}^2 \cdot R_i^{1/4}}{(M_{Pi}^2 - 1)^{3/8}} \right] B = \sum_{i=1}^n \left[\frac{M_{Pi}^2 \cdot R_i^{1/4}}{(M_{Pi}^2 - 1)^{3/8}} T_{Fi} \right] \\ \left[\sum_{i=1}^n \frac{M_{Pi}^2 \cdot R_i^{1/4}}{(M_{Pi}^2 - 1)^{3/8}} \right] K + n \cdot B = \sum_{i=1}^n (M_{Pi} \cdot T_{Fi}) \end{cases}$$

Solve for the undetermined coefficients K and B, then fit a straight line, and then find the corresponding C₁ and C₂.

5. Conclusions

Because the Mach number M_p of linear targatrangle method 1 takes the average of all flight carrier in a certain range calibration test, it will inevitably bring some errors to the calculation of C₁ and C₂. However, linear fitting method 2 uses the Mach number of each flight carrier, and the fitted C₁ and C₂ are more reasonable than linear fitting method 1. For linear fitting method 2 and nonlinear fitting method, C₁ and C₂ obtained by these two fitting methods can be used to calculate the corresponding average relative error between the incitation distance R and the actual value to judge the advantages and disadvantages of these two test methods. Here, a transtmission test data is used to illustrate. Figure 2 shows the comparison table of the average relative error of R under different Mach numbers based on C₁ and C₂ fitted to 600 m incitation data by these two methods.

Figure 2. The error contrast of R under different Mach numbers is obtained by fitting C₁ and C₂ with two methods.

Target Range/m	Incentive	Average Relative Error(%)		M _p
		Linear Fitting Method 2	Nonlinear Fitting Method	
		C ₁ =106.814 C ₂ =82.017	C ₁ =115.914 C ₂ =93.857	
400	Strength	15.68	16.28	2.847
400	Plus	12.01	11.26	2.747
600	Strength	6.92	6.45	2.697
600	Plus	2.67	2.87	2.586
800	Plus	6.89	7.03	2.448
400	Minus	2.33	3.76	2.302
600	Minus	3.81	3.59	2.114
800	Minus	3.59	3.63	2.035

The error comparison table shows that linear fitting method 2 is better. Moreover, when fitting linearly, the fitting points are displayed on the screen graphics with coordinate points. The line resulting from the fit can also be drawn on the screen, so that the distribution of the data for each test can be seen very intuitively. In fact, due to various reasons, there are always a few flight carrier flight disturbances is too large, the Angle between the flight direction and the flight carrier axis increases, so that the flight carrier shock characteristics are distorted greatly, and the data points of this flight carrier often deviate far from the straight line on the graph. Such abnormal

flight carrier data must be eliminated, and then more reasonable values of C_1 and C_2 are fitted again.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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