

Semiconductor Research Based on Liu Enke's "Semiconductor Physics" and Huang Kun's Solid-state Physics

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Abstract:

The semiconductor industry is developing rapidly nowadays, and due to its important position in areas such as chips, it also has great prospects for future development. This research report introduces and explains the application of solid-state physics in the study of semiconductors from four aspects: impurities in semiconductors, statistical distribution of charge carriers in semiconductors, conductivity of semiconductors, and non-equilibrium charge carriers, based on Liu Enke's "Semiconductor Physics" and Huang Kun's materials on solid-state physics.

Keywords:

Solid State Physics, Semiconductor, Impurities, Statistical Distribution, Conductivity, Non-equilibrium

1. Introduction

In the contemporary technological landscape, the semiconductor industry stands as a core driving force behind global technological innovation and industrial upgrading, permeating critical fields such as integrated circuits, artificial intelligence, telecommunications, and new energy. As the fundamental material supporting chip manufacturing and various electronic devices, semiconductors have become a strategic cornerstone for national technological competitiveness and economic development, rendering in-depth research on their intrinsic properties and regulatory mechanisms an urgent and significant task.

Solid-state physics, as a foundational discipline exploring the structural and physical properties of condensed matter, provides an indispensable theoretical framework for deciphering the complex behaviors of semiconductors. The unique electrical, optical, and thermal properties of semiconductors—distinct from conductors and insulators—originate from their specific crystal structures and electron energy band characteristics, which are precisely the core research objects of solid-state physics. Over the past few decades, the integration of solid-state physics theories into semiconductor research has achieved landmark breakthroughs: from the

clarification of intrinsic and extrinsic semiconductor conduction mechanisms to the precise control of charge carrier concentration, solid-state physics has continuously laid the theoretical foundation for the design, preparation, and performance optimization of semiconductor materials and devices.

Against this backdrop, this research report takes the intersection of solid-state physics and semiconductor research as the core entry point, aiming to systematically elaborate on the application of solid-state physics theories in semiconductor studies. Drawing on authoritative works such as Liu Enke's **Semiconductor Physics** and Huang Kun's solid-state physics materials—both of which are recognized as classic references in the field, combining rigorous theoretical systems with practical research insights—this report focuses on four key dimensions. Specifically, it will analyze the influence of impurities on semiconductor energy bands and conductivity based on solid-state physics impurity energy level theory; clarify the statistical distribution laws of electrons and holes in semiconductor energy bands using statistical physics methods in solid-state physics; explore the intrinsic and extrinsic conductivity mechanisms of semiconductors by linking electron transport theories with band structure; and discuss the generation, recombination, and transport characteristics of non-equilibrium charge carriers under external excitation, which are crucial for optoelectronic device applications.

By systematically combing the application of solid-state physics in these four core aspects, this report intends to reveal the intrinsic connection between solid-state physics theories and semiconductor performance regulation, providing a clear theoretical reference for subsequent semiconductor material design, device development, and performance optimization. It also aims to highlight the irreplaceable role of basic physics disciplines in promoting high-tech industrial development, thereby laying a foundation for deeper integration of theoretical research and industrial practice in the semiconductor field.

2. Impurities

Impurities and atoms enter the semiconductor to form defects, which can only occur through gaps and substitutions. V group elements doped into silicon have an excess valence electron, which is weakly bound (0.04 eV) and easily ionized.

This type of impurity is called donor impurity. The binding energy level of the donor impurity is attached below the bottom of the conduction band. Semiconductors that primarily rely on conduction band electrons for conductivity are called n-type semiconductors.

When III group elements are doped into silicon, one less valence electron can be taken from other silicon atoms, resulting in a hole in the silicon and the formation of a negative charge center by impurity atoms. This type of impurity is called acceptor impurity. The acceptor level is located in the bandgap close to the valence band. III. V group impurities in silicon belong to shallow level impurities, and their binding energy can be approximately calculated using a hydrogen like atomic model.

When there are both donor impurities and acceptor impurities, there will be a compensating effect. When other group elements are added, deep energy levels may also be generated. This energy level has little effect on carrier concentration and conductivity type, but has a significant impact on the recombination process, so these energy levels are also known as recombination centers. Shown in Figure 1.

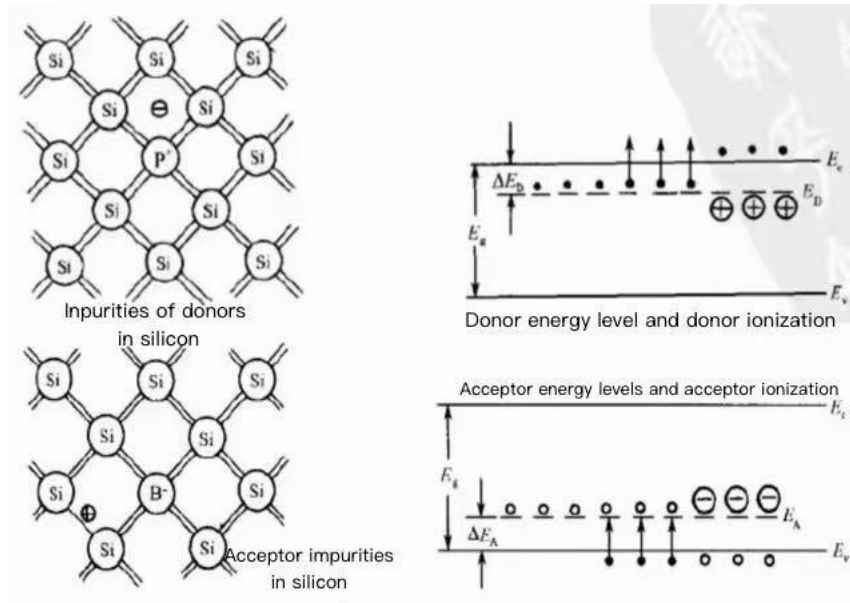


Figure 1. Impurities and energy levels of acceptor and donors [1].

In elemental crystals, point defects such as vacancies and gaps can also be formed. Taking silicon as an example, when vacancies appear, four unsaturated covalent bonds appear, which tend to accept electrons. Therefore, vacancies exhibit an acceptor effect. Similarly, interstitial atoms exhibit donor effects. When the composition deviates from the supported chemical ratio, point defects such as arsenic vacancies and gallium vacancies in gallium arsenide may also occur. The manifestation of this defect as donor or acceptor requires experimental determination. The arsenic vacancies and gallium vacancies in gallium arsenide exhibit acceptor interactions.

Dislocations are also a type of defect in semiconductors that can have a serious impact on material properties. Currently, we only have a better understanding of dislocations in silicon and germanium. Surrounding dislocations, lattice distortion occurs, causing changes in the energy of the conduction and valence bands in the vicinity.

3. Statistical Distribution of Charge Carriers In Semiconductors

3.1. State density

Firstly, calculate the density of states near the bottom of the semiconductor conduction band/top of the valence band. State density is defined as [2]:

$$g(E) = \frac{dZ}{dE} \quad (1)$$

The quantum number of energy intervals per unit energy near E. Near the bottom of the conduction band, its dispersion relationship is : (when the energy surface is spherical)

$$E(k) = E_c + \frac{\hbar^2 k^2}{2m_n^*} \quad (2)$$

So

$$g_c(E) = \frac{dZ}{dE} = \frac{V}{2\pi^2} \frac{(2m_n^*)^{\frac{3}{2}}}{\hbar^3} (E - E_c)^{\frac{1}{2}} \quad (3)$$

The same applies to the top of the price band:

$$g_v(E) = \frac{V}{2\pi^2} \frac{(2m_n^*)^{\frac{3}{2}}}{\hbar^3} (E - E_c)^{\frac{1}{2}} \quad (4)$$

This expression also holds for equipotential surfaces of ellipsoids, but the effective mass is a function of the mass in all directions

3.2. Nondegenerate semiconductor

Electrons in semiconductors follow the Fermi distribution. The probability of a quantum state with energy E being occupied by electrons [2] is

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_0 T}\right)} \quad (5)$$

When $E - E_F \gg k_0 T$, has

$$f(E) = \exp\left(-\frac{E - E_F}{k_0 T}\right) \quad (6)$$

Transform into a Boltzmann distribution. In general semiconductors (referred to as non degenerate semiconductors), the Fermi level is located within the bandgap and is far away from the top/bottom of the conduction band by a distance greater than $k_0 T$. Therefore, the Boltzmann distribution can be used to describe the electrons/holes in it. So the number of electrons in the valence band near E

$$dN = \frac{V}{2\pi^2} \frac{(2m_n^*)^{\frac{3}{2}}}{\hbar^3} \exp\left(-\frac{E - E_F}{k_0 T}\right) (E - E_c)^{\frac{1}{2}} dE \quad (7)$$

Or the number per unit volume:

$$dn = \frac{dN}{V} = \frac{1}{2\pi^2} \frac{(2m_n^*)^{\frac{3}{2}}}{\hbar^3} \exp\left(-\frac{E - E_F}{k_0 T}\right) (E - E_c)^{\frac{1}{2}} dE \quad (8)$$

So the electron density in the valence band: (considered to be very high at the top of the valence band)

$$\begin{aligned} n_0 &= \int dn = \frac{1}{2\pi^2} \frac{(2m_n^*)^{\frac{3}{2}}}{\hbar^3} \exp\left(-\frac{E_c - E_F}{k_0 T}\right) \int_0^\infty x^{1/2} e^{-x} dx \\ &= 2 \left(\frac{m_n^* k_0 T}{2\pi\hbar^2} \right)^{3/2} \exp\left(-\frac{E - E_F}{k_0 T}\right) \\ &= N_c \exp\left(-\frac{E - E_F}{k_0 T}\right) \end{aligned} \quad (9)$$

Among them, $2 \left(\frac{m_n^* k_0 T}{2\pi\hbar^2} \right)^{3/2}$ is called the effective state density of the conduction band

Similarly,

$$\begin{aligned} p_0 &= N_v \exp\left(\frac{E_v - E_F}{k_0 T}\right) \\ N_v &= 2 \left(\frac{m_n^* k_0 T}{2\pi\hbar^2} \right)^{3/2} \end{aligned} \quad (10)$$

We found that

$$\begin{aligned} n_0 p_0 &= N_c N_v \exp\left(-\frac{E_c - E_v}{k_0 T}\right) \\ &= 4 \left(\frac{k_0}{2\pi\hbar^2}\right)^3 \left(m_n^* m_p^*\right)^{3/2} T^3 \exp\left(-\frac{E_g}{k_0 T}\right) \end{aligned} \quad (11)$$

It is independent of the Fermi level and impurities, and only depends on temperature (and bandgap width)

3.3. Intrinsic semiconductor

Intrinsic semiconductor refers to a semiconductor without impurities and defects. In intrinsic semiconductors, there are $n_0 = p_0$, so the Fermi energy can be calculated

$$E_F = \frac{E_c + E_v}{2} + \frac{k_0 T}{2} \ln \frac{N_v}{N_c} = E_i \quad (12)$$

And

$$n_0 = p_0 = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2k_0 T}\right) = n_i \quad (13)$$

Only very pure semiconductors can be considered intrinsic semiconductors. In general semiconductor devices, charge carriers mainly come from impurity ionization, and the contribution of intrinsic excitation can be omitted. But as the temperature increases, the intrinsic carrier concentration rapidly increases. When intrinsic charge carriers dominate, the device cannot function properly.

3.4. Impurity semiconductor

The acceptor/donor energy level can accommodate up to one electron (unlimited spin) and is not allowed to be occupied by two electrons with opposite spin directions at the same time. This is different from the energy levels within the energy band, resulting in a distribution function of impurity energy levels that is also different from the energy band. It can be proven (such as writing about the grand canonical ensemble) the probability of electrons occupying the donor energy level [3]:

$$f_D(E) = \frac{1}{1 + \frac{1}{g_D} \exp\left(\frac{E_D - E_F}{k_0 T}\right)} \quad (14)$$

The electron concentration of the donor energy level is

$$n_D = N_D f_D(E) = \frac{N_D}{1 + \frac{1}{g_D} \exp\left(\frac{E_D - E_F}{k_0 T}\right)} \quad (15)$$

Where N_D is the donor concentration and g_D is the ground state degeneracy of the donor energy level.

It is also possible to calculate the concentration of ionized donors

$$n_D^+ = N_D - n_D = \frac{N_D}{1 + g_D \exp\left(\frac{E_D - E_F}{k_0 T}\right)} \quad (16)$$

When $E_D - E_F \gg k_0 T$, the donor is almost completely ionized.

Consider an n-type semiconductor with only one donor impurity, under the condition of electrical neutrality

$$n_0 = n_D^+ + p_0 \quad (17)$$

Has (let $g_D=2$)

$$N_c \exp\left(-\frac{E_c - E_F}{k_0 T}\right) = N_v \exp\left(\frac{E_v - E_F}{k_0 T}\right) + \frac{N_D}{1 + 2 \exp\left(-\frac{E_D - E_F}{k_0 T}\right)} \quad (18)$$

So we can calculate the Fermi energy. But it's really difficult to solve, so we can only approximate the temperature zones and see the pattern of Fermi energy changes.

According to the degree of impurity ionization and intrinsic excitation, the entire temperature range is divided into five parts: “low-temperature weak ionization zone”, “intermediate ionization zone”, “strong ionization zone”, “transition zone”, and “high-temperature intrinsic excitation zone”.

3.4.1. Low temperature weak ionization zone

At this time, the temperature in the low-temperature weak ionization zone is very low, and only a small amount of donor impurities undergo ionization [3], while intrinsic excitation is even less. So there is $n_0 = n_D^+ \ll N_D$, that is

$$\begin{aligned} N_c \exp\left(-\frac{E_c - E_F}{k_0 T}\right) &= \frac{N_D}{1 + 2 \exp\left(-\frac{E_D - E_F}{k_0 T}\right)} \\ &= \frac{1}{2} N_D \exp\left(\frac{E_D - E_F}{k_0 T}\right) \end{aligned} \quad (19)$$

Logarithmic simplification, has

$$E_F = \frac{E_c + E_D}{2} + \left(\frac{k_0 T}{2}\right) \ln\left(\frac{N_D}{2N_c}\right) \quad (20)$$

And electron concentration

$$n_0 = \left(\frac{N_c N_D}{2}\right)^{1/2} \exp\left(-\frac{E_c - E_D}{2k_0 T}\right) \quad (21)$$

It can be seen that at this point, the Fermi energy first increases and then decreases with temperature, while the electron concentration increases with temperature.

3.4.2. Intermediate ionization zone

The part located between the strong ionization zone and the low temperature weak ionization zone. As the temperature increases, impurities gradually ionize and the Fermi level gradually decreases.

3.4.3. Strong ionization zone

Most of the impurities here have been ionized, so there are $n_0 \approx n_D^+ \approx N_D$

So

$$N_c \exp\left(-\frac{E_c - E_F}{k_0 T}\right) = N_D \quad (22)$$

And

$$E_F = E_c + k_0 T \ln\left(\frac{N_D}{N_c}\right) \quad (23)$$

When all donor impurities are ionized and intrinsic excitation can be ignored, the electron concentration is independent of temperature and is called the saturation region

3.4.4. Transition zone

The part located between the strong ionization zone and the high-temperature intrinsic excitation zone is called the transition zone. All impurities here have been ionized, and intrinsic excitation cannot be ignored, with $n_0 = N_D + p_0$. It can be solved:

$$E_F = E_i + k_0 T \operatorname{arcsch} \left(\frac{N_D}{2n_i} \right) \quad (24)$$

Meanwhile $n_0 p_0 = n_i^2$, then we have

$$n_0 = \frac{N_D}{2} \left[1 + \left(1 + \frac{4n_i^2}{N_D^2} \right)^{1/2} \right] \sim N_D + \frac{n_i^2}{N_D} \quad (25)$$

$$p_0 = \frac{n_i^2}{n_0} \sim \frac{n_i^2}{N_D} \quad (26)$$

The final approximation is effective when $n_i \ll N_D$, where there is a significant concentration difference between electrons and holes, and the semiconductor is located near the saturation region. The concentration of minority carriers (in this case, holes) rapidly increases with temperature. And when $n_i \gg N_D$, we have

$$\begin{aligned} n_0 &= \frac{N_D}{2} + n_i \\ p_0 &= -\frac{N_D}{2} + n_i \end{aligned} \quad (27)$$

At this point, the electron and hole concentrations are similar, and the semiconductor is in a position close to intrinsic excitation.

3.4.5. High temperature intrinsic excitation zone

As the temperature in the high-temperature intrinsic excitation region continues to rise, the concentration of carriers excited by the intrinsic excitation rapidly increases, far exceeding the impurity concentration. At this point, the electrical neutrality condition is the same as that of the intrinsic semiconductor, and the Fermi level also moves to the vicinity of the bandgap centerline. The following Figure 2 shows the relationship between electron concentration and temperature in n-type silicon, which clearly shows the transition from low temperature weak ionization saturation to high temperature intrinsic excitation.

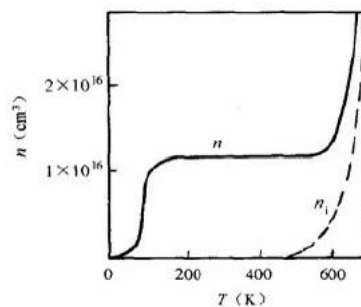


Figure 2. The curve of electron concentration and temperature for n-type silicon [1].

For p-type semiconductors and semiconductors with both donor and acceptor impurities, the discussion method is the same as above, and can ultimately be divided into the five temperature zones mentioned above.

3.5. Degenerate semiconductors

Taking n-type semiconductors as an example, in the low-temperature weak ionization region, the Fermi level first rises and then falls, and there is a possibility that the Fermi level may enter the valence band. At this point, the original method of using Boltzmann distribution to describe the distribution of electrons in the conduction band is no longer effective. Semiconductors with Fermi levels outside the bandgap are called degenerate semiconductors, and those with high impurity concentrations are called heavily doped [2].

$$n_0 = \frac{(2m_n^*)^3}{2\pi^2\hbar^3} \int_{E_c}^{\infty} \frac{(E-E_c)^{1/2}}{1+\exp\left(\frac{E-E_F}{k_0T}\right)} dE \quad (28)$$

Let

$$\int_0^{\infty} \frac{x^{1/2}}{1+\exp(x-\zeta)} = F_{1/2}(\zeta) \quad (29)$$

We can get

$$n_0 = N_c \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_F-E_c}{k_0T}\right) \quad (30)$$

It can be considered that the degeneracy effect needs to be taken into account when

$$E_F \geq E_c - 2k_0T \quad (31)$$

In degenerate semiconductors, when the impurity concentration is high, the impurity energy levels will expand into energy bands, allowing electrons to participate in conduction. If the impurity band is connected to the original conduction band, the bandgap width will narrow. This is called the bandgap narrowing effect. Shows in Figure 3.

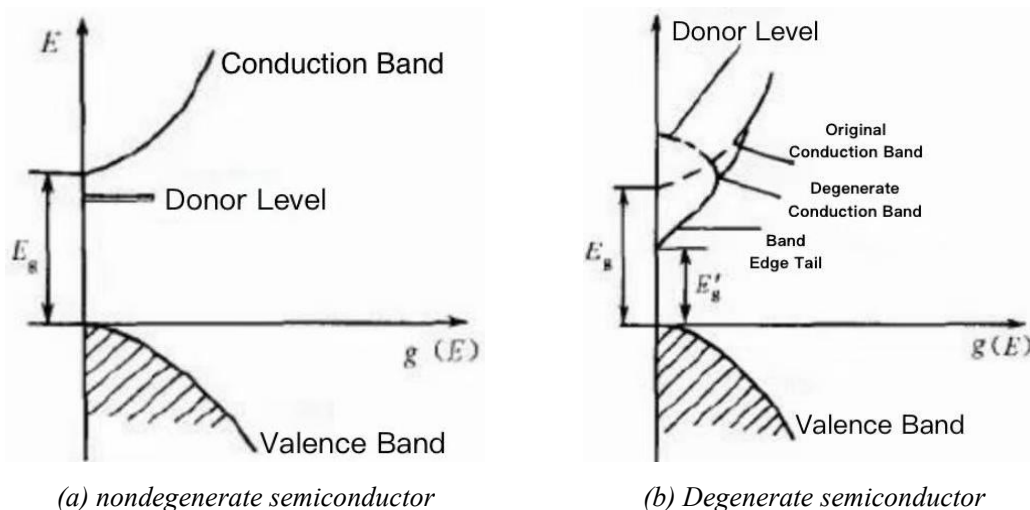


Figure 3. The relationship between state density $g(E)$ and energy E [1].

4. The Conductivity of Semiconductors

4.1. Basic concepts

We have Ohm's law and its differential form to describe the drift motion of charge carriers under the action of an electric field and the formation of current:

$$I = \frac{V}{R} \quad j = \sigma E \quad (32)$$

The ratio of average drift velocity to electric field is also referred to as mobility:

$$\mu = \left| \frac{\bar{v}_d}{E} \right| = \frac{\sigma}{nq} \quad (33)$$

In semiconductors, there are two types of charge carriers: electrons and holes. These two types of charge carriers have their own mobility and concentration, and contribute to conductivity respectively.

4.2. Scattering of charge carriers

The scattering of carriers has a significant impact on the drift process of carriers. When there is an external electric field, charge carriers accelerate under the action of the electric field. Scattering prevents the acceleration process of charge carriers from continuing indefinitely.

(Note: This section of the book probably did not consider the matter of energy bands. Under the action of an electric field, charge carriers move uniformly in k-space. It is scattering that prevents this uniform motion from continuing, resulting in a constant current. See any solid state physics book for details.)

The fundamental reason for charge carrier scattering is that the periodic potential field is disrupted, allowing for transitions between states of different k. The common causes of scattering include ionization impurity scattering, lattice vibration scattering (phonon scattering), and other factors (valley scattering, neutral impurity scattering, dislocation scattering, alloy scattering, etc.). There is also scattering between charge carriers during strong degeneracy.

4.2.1. The relationship between mobility, impurity concentration, and temperature: a simple model

Below, I attempt to discuss the related issues of mobility through a simple model that does not consider the statistical distribution of carrier velocity.

If there are $N(t)$ electrons moving in a certain direction at a velocity v and the scattering probability is P , then there are

$$\frac{dN}{dt} = -PN \quad (34)$$

The same with

$$N(t) = N_0 \exp(-Pt) \quad (35)$$

The number of electrons in the free time range $t \sim (t+dt)$ is $N_0 P \exp(-Pt) dt$, so the average free time

$$\tau = \frac{1}{N_0} \int_0^\infty N_0 P \exp(-Pt) dt = \frac{1}{P} \quad (36)$$

Now apply an electric field E in the x direction, and consider a certain electron scattered at $t = 0$, assuming its velocity in the x direction after scattering is v_{x0} , then the velocity in the x direction before scattering again is

$$v_x = v_{x0} - \frac{q}{m_n^*} Et \quad (37)$$

Considering its average value and assuming $\overline{v_{x0}} = 0$ there is

$$\begin{aligned} \overline{v_x} &= \overline{v_{x0}} - \int_0^\infty \frac{q}{m_n^*} Et P \exp(-Pt) dt \\ &= -\frac{qE}{m_n^*} \tau_n \end{aligned} \quad (38)$$

So

$$\mu_n = \frac{|\overline{v_x}|}{E} = \frac{q\tau_n}{m_n^*} \quad (39)$$

Similarly, for holes, there is

$$\mu_p = \frac{q\tau_p}{m_p^*} \quad (40)$$

For multiple energy valleys, each with a different effective mass, the final expression is similar to the above, except that the effective mass is replaced with “conductivity effective mass”.

As mentioned above, there are many scattering mechanisms. Generally speaking, these mechanisms exist simultaneously. Assuming the scattering probability of the i th mechanism is P_i , and the corresponding average free time is $\tau_i = \frac{1}{P_i}$, there are total scattering probability and total free time:

$$P = \sum_i P_i \quad \tau^{-1} = \sum_i \tau_i^{-1} \quad (41)$$

For the scattering mechanism mentioned earlier, the relationship between its average free time and parameters such as temperature is given here in Figure 4 without further proof.

Scattering mechanism	Mean free time
Ionized impurity scattering	$\tau_i \propto N_i^{-1} T^{3/2}$
Acoustic wave scattering	$\tau_s \propto T^{-3/2}$
Optical wave scattering	$\tau_0 \propto \left[\exp\left(\frac{\hbar\omega_l}{k_0 T}\right) - 1 \right]$

Figure 4. Relationship between its average free time and parameters.

4.2.2. Impurity concentration

Next, let's focus on the relationship between the mobility of multiple and few carriers and impurity concentration.

Impurity concentration ($N = N_A \text{ or } N_D$)

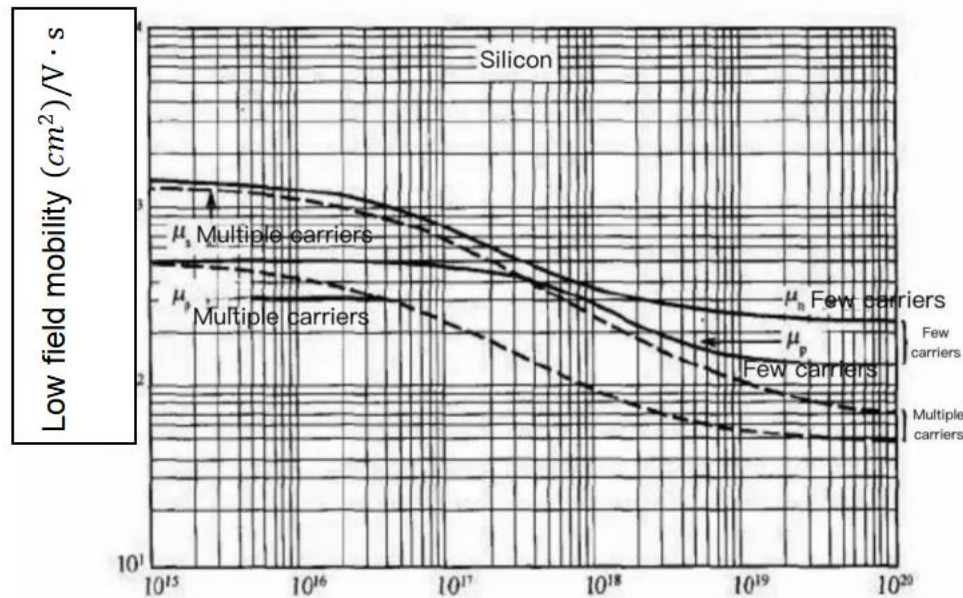


Figure 5. The solid line shows the relationship between the majority carrier and minority carrier mobility and impurity concentration in n-type and p-type silicon at room temperature: Solid line Minority carrier mobility; Dashed line: Majority carrier mobility [1].

It can be seen in Figure 5 that :

- (1). when the impurity concentration is low, the mobility of multi carriers and few carriers for the same type of carrier is almost equal.
- (2). When the impurity concentration increases, the mobility monotonically decreases.
- (3). For the same type of carrier, the mobility of minority carriers is greater than that of majority carriers, and the difference between the two increases with the increase of impurity concentration. This is because impurities can capture multiple molecules, thereby reducing their mobility.

4.2.3. Temperature

For pure semiconductor materials, the resistivity is mainly determined by the carrier concentration. The carrier concentration increases sharply with temperature, resulting in a decrease in the resistivity of intrinsic semiconductors with increasing temperature. This is different from metal.

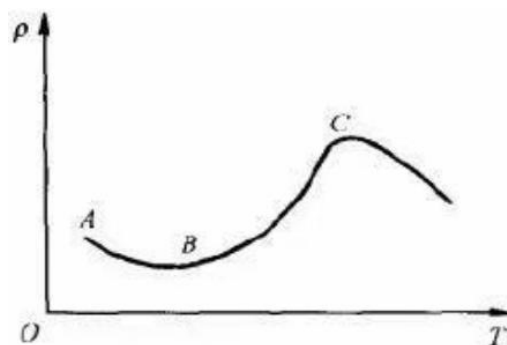


Figure 6. Schematic diagram of the relationship between silicon resistivity and temperature [1].

Impurity semiconductors are more complex. The carrier concentration is determined by two processes: impurity ionization and intrinsic excitation, and the

scattering mechanism also includes ionization impurity scattering and lattice scattering.

As shown in the above Figure 6, the resistivity temperature curve is mainly divided into three sections:

(1) The temperature in section AB is very low, and the dominant mechanism is impurity ionization providing charge carriers and ionizing impurity scattering. The resistivity decreases with increasing temperature. (As the temperature rises, the concentration of ionized impurities increases. Why is it referred to as an increase in mobility in the book?)

(2) The impurities in the BC segment are completely ionized, and the intrinsic excitation is not significant. The carrier concentration changes little, but the scattering process is dominated by phonon scattering, resulting in an increase in resistivity.

(3) After C, the temperature becomes higher, and intrinsic excitation plays a dominant role. At this point, the performance of the semiconductor is similar to that of an intrinsic semiconductor, with the resistivity decreasing with increasing temperature.

Generally speaking, when the temperature rises to the point where intrinsic excitation dominates, the device cannot function properly. This temperature is generally 100 °C for germanium devices, 250 °C for silicon devices, and 450 °C for gallium arsenide devices.

4.3. Boltzmann equation and statistical theory of conductivity

When there is an external electric field (or temperature gradient), the distribution function of electrons will change from the original f_0 to a new $f(k, r, t)$. If the new distribution function is obtained, the current can be calculated, and then the conductivity can be calculated. Next, try to calculate the equation that the new distribution function satisfies.

4.3.1. Boltzmann equation

The number of electrons in the phase space volume element with wave vector at $k \sim k+dk$ and potential vector at $r \sim r+dr$ at time t [2] is

$$dN(k, r, t) = 2f(k, r, t) dk dr \quad (42)$$

After dt time, there is

$$\begin{aligned} dN(k, r, t+dt) &= 2f(k, r, t+dt) dk dr \\ &= 2 \left[f(k, r, t) + \frac{\partial f}{\partial t} dt \right] dk dr \end{aligned} \quad (43)$$

The change in the number of electrons is caused by the variation of the distribution function over time. There are two main reasons for changes in the distribution function: drift caused by external fields and scattering effects.

The changes caused by drift variations are represented by the subscript d , which includes

$$\left(\frac{\partial f}{\partial t} \right)_d = -(\dot{k} \cdot \nabla_k f + \dot{r} \cdot \nabla_r f) \quad (44)$$

The changes caused by scattering are represented by the subscript s , with a rate of change of $\left(\frac{\partial f}{\partial t} \right)_s$

The total change is the sum of the two mentioned above

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_s - (\dot{k} \cdot \nabla_k f + v \cdot \nabla_r f) \quad (45)$$

In steady state, there is $\frac{\partial f}{\partial t} = 0$, so

$$\left(\frac{\partial f}{\partial t} \right)_s = \dot{k} \cdot \nabla_k f + v \cdot \nabla_r f \quad (46)$$

The above equation is called the Boltzmann equation. In a spatially uniform system, the above equation can be simplified as

$$\left(\frac{\partial f}{\partial t} \right)_s = \dot{k} \cdot \nabla_k f \quad (47)$$

4.3.2. Relaxation time approximation

With the Boltzmann equation, we can theoretically derive a new distribution function. But this is difficult because we don't yet know the specific form of the scattering term. The relaxation time approximation can simplify the problem.

Assuming that electrons only move freely for a short period of time τ , causing the distribution function to change from zero field f_0 to steady-state f with added field, and then be re scattered to the state of f_0 . So there is

$$\begin{aligned} f(k, \tau) &= f_0(k - \dot{k}\tau) \\ &\approx f_0 - \tau \dot{k} \cdot \nabla_k f_0 \\ &\approx f_0 - \tau \dot{k} \cdot \nabla_k f \end{aligned} \quad (48)$$

Furthermore, we can obtain

$$\left(\frac{\partial f}{\partial t} \right)_s = -\frac{f - f_0}{\tau} \quad (49)$$

Now we add a small electric field E to the system, which has

$$\dot{k} = -\frac{qE}{\hbar} \quad (50)$$

Introduce the Boltzmann equation

$$-\frac{qE \cdot \nabla_k f}{\hbar} = -\frac{f - f_0}{\tau} \quad (51)$$

Let $f = f_0 + \phi$, where ϕ is very small, obtain

$$\begin{aligned} \phi &= \frac{q}{\hbar} \tau \nabla_k (f_0 + \phi) \cdot E \\ &\approx \frac{q}{\hbar} \tau \nabla_k f_0 \cdot E \\ &= \frac{q}{\hbar} \tau \frac{\partial f_0}{\partial E} \nabla_k E \cdot E \\ &= q \tau \frac{\partial f_0}{\partial E} v \cdot E \end{aligned} \quad (52)$$

Therefore, the current density is:

$$J = -2q \int v f dk = -2q \int v \phi dk = -2q^2 \int \tau \frac{\partial f_0}{\partial E} v (v \cdot E) dk \quad (53)$$

So the conductivity tensor is

$$\sigma_{ij} = -2q^2 \int \tau \frac{\partial f_0}{\partial E} v_i v_j dk \quad (54)$$

For isotropic scattering and spherical equipotential surfaces, $\tau \frac{\partial f_0}{\partial E}$ is direction independent, so

$$\sigma_{ij} = \sigma \delta_{ij}, J = \sigma E \quad (55)$$

Further

$$\begin{aligned} \sigma &= (\sigma_{11} + \sigma_{22} + \sigma_{33})/3 \\ &= -\frac{2}{3} q^2 \int \tau \frac{\partial f_0}{\partial E} v^2 dk \\ &= \frac{q^2}{3k_0 T} \int 2 \tau f_0 v^2 dk \\ &= \frac{q^2}{3k_0 T} \int \tau v^2 dn \end{aligned} \quad (56)$$

(For non degenerate semiconductors, f_0 is a Boltzmann distribution)

As can be seen, we expressed the conductivity using the parameter of relaxation time, which is good. But we still don't know the specific expression for relaxation time. To understand this, a detailed and specific analysis of the scattering process is required.

(An example is phonon scattering, which can be referred to in the “Lattice Scattering and Conductivity” section of Huang Kun's Solid State Physics, omitted here.) [2]

4.4. Strong electric fields

When the electric field is strong, semiconductors will deviate from Ohm's law and their conductivity will no longer be constant. The conductivity depends on the carrier concentration and average drift velocity. Experiments have found that under a certain electric field, when the carrier concentration does not change, there will be a deviation from Ohm's law. This indicates that the average drift velocity is no longer proportional to the electric field. This is shown in Figure 7 below.

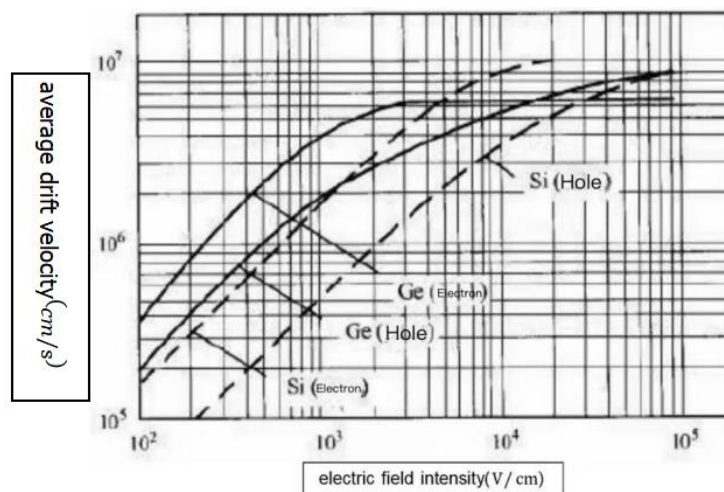


Figure 7. The relationship between the average drift velocity of germanium and silicon and the electric field strength (300K) [1].

In the case of a weak electric field, charge carriers obtain energy from the electric field and then transfer the energy to the lattice, that is, emit phonons. At this point, there is thermal equilibrium between the charge carriers and the lattice. In the case of a strong electric field, carriers gain a lot of energy and are no longer in thermal equilibrium with the lattice. Such carriers are called hot carriers. We introduce the effective temperature T_e of charge carriers to describe such carriers. Hot carriers have high energy, resulting in a shorter average free time and lower mobility.

4.5. Multi energy Valley Scattering and Gunn Effect

As shown in the Figure 8 & Figure 9, in gallium arsenide, the lowest valley 1 and the valence band top are located at $k=0$. In addition, point L also has a conduction band valley 2. The effective mass of valley 2 is greater than that of valley 1. When the temperature is not high and the electric field is not strong, most of the electrons are in energy valley 1. However, when the electric field reaches a certain strength, n-type gallium arsenide will exhibit negative differential conductivity. This is because under high electric fields, electrons can gain enough energy to transfer from valley 1 to valley 2 with the assistance of phonons. The effective mass of Nenggu 2 is large, so the migration rate is relatively small. Therefore, this transfer (known as multi valley scattering) can lead to negative differential conductance.

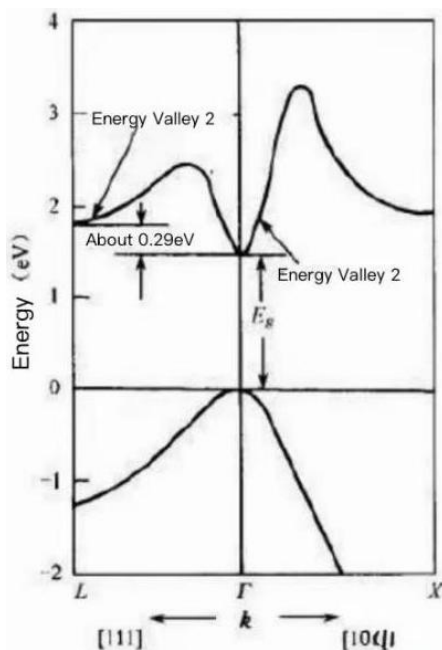


Figure 8. Gallium Arsenide Band Structure [1].

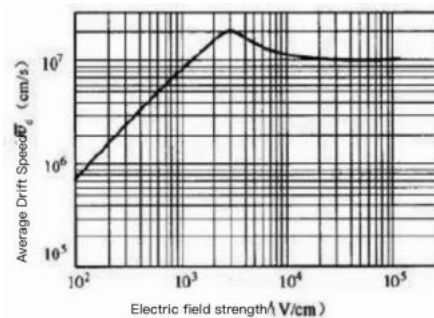


Figure 9. The relationship between the average drift velocity of gallium arsenide electrons and the electric field strength (300K) [1].

The use of this effect can generate microwave oscillations, known as Gunn oscillations. Apply voltage across an n-type gallium arsenide block to put it in a negative differential mobility state. If there is unevenness inside the device and a small amount of space charge is generated, this charge will rapidly grow into a dipole domain until the electric field inside the domain causes the material to enter a positive differential mobility state. At this point, the domain reaches a stable state and moves towards the anode at a constant speed. After the domain moves to the anode, it will be absorbed and generate voltage fluctuations, while a new domain begins to form. The

oscillation frequency of the device depends on the rate of repeated domain formation. Generally speaking, the shorter the device length, the higher the oscillation frequency.

5. Nonequilibrium carriers

5.1. Basic concepts

A semiconductor in thermal equilibrium has a constant carrier concentration at a certain temperature. But if we apply external forces to it, such as electricity or light. Taking illumination as an example, as long as the energy of the incident photon is greater than the bandgap width of the semiconductor, the photon will excite valence band electrons into the conduction band, producing electron hole pairs. At this point, the carrier concentration is no longer in equilibrium. These extra electrons and holes are called non-equilibrium carriers, and their concentrations are represented by Δn (electrons) and Δp (holes). For light injection, $\Delta n = \Delta p$.

The situation where the injected non-equilibrium carrier concentration is much lower than the equilibrium multi carrier concentration is generally referred to as small injection. Even in the case of small injection, the non-equilibrium minority carrier concentration can be much higher than the equilibrium minority carrier concentration, which has a significant impact on semiconductor properties. The commonly referred non-equilibrium carriers refer to non-equilibrium minority carriers.

When the external effect is removed, the injected non-equilibrium carriers will gradually disappear, and the semiconductor will return to its original equilibrium state. This process is called non-equilibrium carrier recombination. Taking light injection as an example, experiments have shown that after the external effect is removed, the non-equilibrium carrier concentration decreases exponentially. The average survival time is referred to as the lifetime of non-equilibrium carriers, denoted as τ . Due to the dominant influence of non-equilibrium minority carriers in non-equilibrium carriers, the lifetime of non-equilibrium carriers is often referred to as the minority carrier lifetime.

Assuming that non-equilibrium carriers Δn_0 and Δp_0 are uniformly generated through light injection in an n-type semiconductor. At $t=0$, the light suddenly stops, so there is

$$\frac{d\Delta p}{dt} = -\frac{\Delta p}{\tau} \quad (57)$$

It can be verified that this is an equation that exponentially decays the non-equilibrium minority carrier concentration with an average lifetime of τ .

When a semiconductor is in equilibrium, both electrons and holes can be described by Fermi distribution. When external influences disrupt the thermal equilibrium, we can assume that the relaxation time of thermal equilibrium within the conduction and valence bands is short, while the relaxation time of thermal equilibrium between bands is long. So electrons and holes can be described by distribution functions with different Fermi levels. These two Fermi levels are called “quasi Fermi levels”, and the quasi Fermi level of electrons (holes) is represented by E_{Fn} (E_{Fp}). Generally speaking, the quasi Fermi level of many carriers is not much different from the Fermi level at equilibrium, while the quasi Fermi level of few carriers is much different from the equilibrium Fermi level.

5.2. The motion of charge carriers

5.2.1. Diffusion

Now we are starting to consider the motion of non-equilibrium charge carriers. Firstly, the presence of non-equilibrium carriers allows for uneven spatial distribution of carrier concentration, leading to carrier diffusion. The flow density caused by diffusion is called diffusion flow density, represented by S_p (holes) and S_n (electrons). Taking n-type semiconductors as an example, the minority carriers are holes, and there are (one-dimensional cases)

$$S_p = -D_p \frac{d\Delta p}{dx} \quad (58)$$

Consider its stability situation:

$$\frac{\Delta p}{\tau} + \frac{dS_p}{dx} = 0 \quad (59)$$

So

$$D_p \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{\tau} \quad (60)$$

Its solution is

$$\Delta p = A \exp\left(-\frac{x}{L_p}\right) + B \exp\left(\frac{x}{L_p}\right) \quad (61)$$

Among them, $L_p = \sqrt{D_p \tau}$ is called the diffusion length. When the length of the semiconductor is much smaller than the diffusion length, the expression of the above solution can be transformed into a linear function of position.

For three-dimensional situations, there are

$$D_p \nabla^2 (\Delta p) = \frac{\Delta p}{\tau} \quad (62)$$

We can use this equation to calculate some more complex situations, such as injecting non-equilibrium holes through a probe that is trapped in a hemispherical shape on the semiconductor surface. In this case, the diffusion efficiency at the injection site is greater than in the one-dimensional case, as the radial outward movement itself will reduce the concentration. Which is shown in Figure 10.

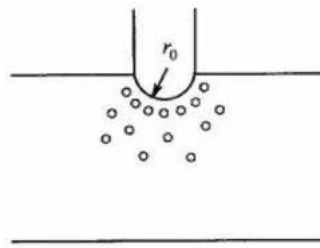


Figure 10. Probe injection [1].

5.2.2. Diffusion & Drift

Now we are joining the drift motion. In the case of one-dimensional uniform semiconductors, we divide the total current density into two parts: drift contribution and diffusion contribution [3]:

$$\begin{aligned} J_p &= qp\mu_p \varepsilon - qD_p \frac{d\Delta p}{dx} \\ J_n &= qn\mu_n \varepsilon + qD_n \frac{d\Delta n}{dx} \end{aligned} \quad (63)$$

Now consider an n-type semiconductor in thermal equilibrium but with non-uniform doping, where the donor impurity concentration is a decreasing function of x . So the total current of electrons and holes should be 0.

So

$$-D_n \frac{dn}{dx} = n\mu_n \varepsilon = -n\mu_n \frac{dV}{dx} \quad (64)$$

And in a state of equilibrium

$$n = N_c \exp \left[\frac{E_F + qV(x) - E_c}{k_0 T} \right] \quad (65)$$

So there is

$$\frac{D_n}{\mu_n} = \frac{k_0 T}{q} \quad (66)$$

Similarly, replacing n with p also holds true. The above equation is called the Einstein relation. Although this formula was derived using balanced charge carriers, experiments have shown that it can also be applied to non-equilibrium charge carriers. This indicates that non-equilibrium carriers are no different from equilibrium carriers for the majority of the time before recombination.

5.2.3. Continuity equation

Next, we will consider the transient process and discuss the equation satisfied by the minority carrier concentration when diffusion and drift coexist. Taking one-dimensional n-type semiconductors as an example, the electric field is along the x -direction. We have continuity equations

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \varepsilon \frac{\partial p}{\partial x} - \mu_p p \frac{\partial \varepsilon}{\partial x} - \frac{\Delta p}{\tau} + g_p \quad (67)$$

The first term on the right side of the equal sign is the contribution of diffusion, the second and third terms are the contributions of drift, the fourth term is the contribution of recombination, and the last term is the concentration change caused by other reasons.

The three-dimensional continuity equation is as follows, with only minor modifications, where J_p is the current density contributed by holes

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot J_p - \frac{\Delta p}{\tau} + g_p \quad (68)$$

The continuity equation reflects the motion law of minority carriers and is one of the fundamental equations for studying the principles of semiconductor devices. If interested, you can give some simple examples, such as the attenuation of non-equilibrium carriers in the absence of an electric field, the drift of minority carrier pulses in an electric field, and the spatial distribution of minority carrier steady states under stable surface illumination.

6. Conclusions

This report carefully explores the important physical principles that determine semiconductor behavior; impurity effects, carrier statistical distribution, conductivity, and nonequilibrium carrier behavior. The investigation explains how impurity doping changes carrier concentrations and Fermi level positions, how nondegenerate and degenerate cases control carrier statistics, and how the scattering mechanisms impact

mobility and conductivity in cases where field and temperature context is important. The bulk of this discussion on nonequilibrium carriers also shows how diffusion, drift, recombination, as well as the continuity equation, are foundational to understanding device operation. As a whole, these principles highlight that solid state physics provides an essential theoretical framework for understanding and constructing contemporary semiconductor devices.

Although this work covers an extensive range of materials and structures, it is still largely theoretical and uses mostly classic models such as the effective mass approximation, Boltzmann distribution model, and relaxation-time approximation. Future work should utilize more sophisticated numerical simulations and quantum-mechanical models, and perform experimental validation of carrier transport predictions, particularly at high field, high-temperature, and nanoscale conditions. From an applications standpoint, refinement in defect-engineering principles, improved mobility-enhancement methods, and better control over non-equilibrium will also provide better performance from semiconductor materials and devices.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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